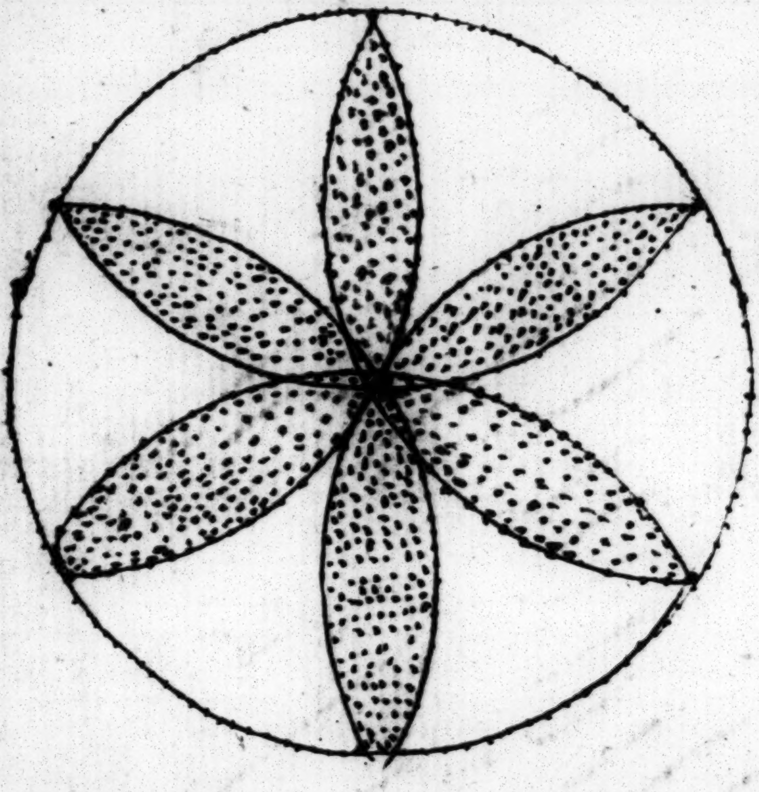

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By DANIEL FENNING, Author of the *Young Algebraist's Companion, Description* and *Use of the* GLOBES, both in Dialogues; the *Universal Spelling-Book, Royal English Dictionary*, and *Young Man's Book of Knowledge.*

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DEDICATION

To the SCHOOLMASTERS of

Great-Britain and *Ireland*.

GENTLEMEN,

THE great Demand for, and ſpeedy Sale of the five laſt Impreſſions of this ſmall Treatiſe, naturally leads me to think, that many of you have encouraged the Undertaking.

Permit me, therefore, in this ſixth Edition, to return you hearty Thanks for your friendly and undeſerved Favours.

I am very ſenſible, (as I told you before) that many abler Hands have undertaken this Taſk before me, and have, in large Volumes, done *that* which cannot be expected to be found here: But as you are ſenſible too many of them have ſpent their Time, rather upon Curioſities than Buſineſs, (the very Thing

that every Scholar ſhould particularly drive at, and which, I am ſenſible, every one of you would promote the Knowledge of) I have here made it my chief Care to inform the Learner of every Thing that is neceſſary thereto.

It was not without Reluctance that I appeared upon this Subject; but having publiſhed a ſmall Treatiſe of FRACTIONS and ALGEBRA, by Way of Dialogue, in 1759, * which has met with great Encouragement, I was perſuaded by ſeveral Schoolmaſters, and private Gentlemen, to publiſh alſo a Piece of ARITHMETIC, after the ſame Manner; as they were ſenſible, they ſaid, (and indeed, I confeſs, I think the ſame) that this Way of Writing conveys the Matter better, and communicates Things ſooner to the Learner, than the bare ſetting of Sums, and not working them at all, or, in a dry, intricate Manner.

As I do not pretend to recommend the Work by comparing it with other Authors; ſo, I hope, you will not con-

* The third Edition, with ſome Quadratic Problems, was publiſhed, 1759.

demn

demn it, till you have perused it throughout.

'Tis true, that the last Method of reducing *C's.* *qrs.* *lbs.* into *lbs.* (in the 1st Example of *Tare* and *Tret*) is well worthy your Observations, it being as easy as it is new; and several Persons in Business, to whom I have communicated it, never pretend to use any other Way for Ease and Expedition.

To such of you, indeed, as are perfect Masters of the Sciences, this little Piece may appear insignificant; but to others, I am sensible, the Dialogues will be of great Service, as they are intermixed with a Variety of familiar Examples, in natural Conversation: And I persuade myself, they will be of great Help to your Pupils, and, in a great Measure, ease you of that heavy Task, which every diligent Master (with a Number of Scholars) must of Necessity labour under.

This, Gentlemen, is one Reason (as I said) why I built it upon this Plan; and I hope, for the Design's Sake, you will forgive those Errors that have yet escaped my Notice, and which you know are so

common to a Work of this Sort; tho' I have taken Care to correct all ſuch, as I have at preſent diſcovered in former Editions.

I have made no great Alteration in the Work, only have taken off Part of the Double Rule of Three *Direct* and *Inverſe*, (as you know the Rule of 5 Numbers will ſupply that Defect) in Order to make more Room (by the Deſire of ſeveral Merchants) for a fuller and more clear Explanation, both of the true and cuſtomary Way of Diſcount.

To my Thanks, I add my beſt Wiſhes for your Succeſs, jointly and ſeparately, in your ſeveral Undertakings; and deſire always to ſubſcribe myſelf,

GENTLEMEN,

Your moſt obliged humble Servant,

Royal-Exchange Aſſurance-Office,
London, Sept. 1,
1767.

D. Fenning.

PRE-

THE

PREFACE.

KIND READER,

I Here present you with the sixth Edition of my small Treatise of ARITHMETIC, which I have improved and endeavoured to render as plain as Time, Room, and Opportunity would admit of.

For such of you, who have too much neglected this Branch of Education; and for others, who have neither Time, nor Opportunity, to apply to a proper Master, the following Work was chiefly designed; and was at first carried on no farther than the *Rule of Three Direct*: But I considered with myself, that it might fall into the

Hands of many, who would be glad to have a Notion of *Fractions*. I have, therefore, to ſerve you, treated upon moſt of the Rules of *Arithmetic*, and for the Sake of thoſe that live in the Country in particular, I have given ſome Inſtructions to meaſure Timber, or a common Piece of Ground; as alſo, to gauge a Caſk, or Piece of Malt, and ſeveral other uſeful and practical Examples, which, I am perſuaded, will be of Service to you.

One Thing, which is ſeldom taken Notice of in a PREFACE, I would have every one of you, that are Learners, to obſerve, and that is, That if, upon the Trial of any Sum, or Queſtion, you do not find it the ſame as the Anſwer, do not let that diſcourage you; for if you examine your own Work after you have done it, and are ſo much Maſter of the Rules as to know when you are right or wrong, you may conclude you are right, whatever Anſwer you may find in my Work, or any other Author's: But before you determine this, mind and be ſure your own Work is right.

As to ſuch of you that have little or no Notion of Figures, if you have a Mind to learn, I am ſenſible you may very eaſily do it, with little or no Trouble; for I have taken the more Pains, that you might have the leſs: And if it does but anſwer the End of inſtructing ſuch of you as are quite ignorant, and improving others that have already ſome Knowledge of Figures, I ſhall look back upon the Undertaking with Pleaſure, notwithſtanding thoſe mean Criticiſms and Remarks, that may be made upon ſuch Overſights, which can hardly be avoided in a Work of this Sort.

But, every impartial Reader will conſider the Nature

Nature and Design of the Work: For 'tis the Plan upon which it is built that is to be minded: If *this* be plain and easy (as I hope it is) there is no Fear but the Learner will find a sensible Satisfaction, and the Work be crowned with Success. As for such *Carpers* that are resolved to amuse themselves with the Bone only, they are extremely welcome; but let them be civil, and not snarl at those who would eat the Flesh quietly. I am,

Kind READER,

Your humble Servant,

and Well-wisher,

London, Sept. 1, 1767.

Daniel Fenning.

TO THE

PUBLIC.

WE whose Names are hereunto subscribed, having perused the Plan of this little Treatise of ARITHMETIC, do allow the Dialogues to be very well adapted to the Purpose: And as the Rules and Examples are laid down in a natural, easy, and familiar Manner, WE beg Leave to recommend it, as the most useful and easy Book for Learners extant.

Edmund Anguish, *Accomptant*
James Barclay, *Writing Master*
William Bently, *Surveyor*
George Coles, *Ditto*
Mr. Coulthist, of the Academy, *Prescot-street, Goodman's-fields*
Henry Deacon, *Accomptant*
Randal Evans, *Writing-Master*
Anthony Gilbert, *Surveyor*
Edward Griffiths, *Ditto*
Samuel Hill, *Philom.*
Thomas Humphreys, *Writing-Master*
Thomas Hughes, *Ditto*
Samuel Hornby, *Ditto*
Timothy Langley, *Accomptant*
Abraham Longden, *Ditto*
Abraham De Lire, *Philom.*
Thomas Newberry, *Ditto*
John Quant, *Writing-Master*
William Richardson, *Ditto*

David

David Rowland, *Accomptant*
John Smyth, *Writing-Maſter*
Jomas Smith, *Ditto*
Thomas Smithe, *Ditto*
John Smythe, *Accomptant*
Zachary Snaper, *Ditto*
Eraſmus Turner, *Ditto*
Johnſon Thompſon, *Writing-Maſter*
James Thurſton, *Ditto*
Daniel Trunker, *Ditto*
William Thorley, *Ditto*
James Thorpe, *Ditto*

To the RECOMMENDERS.

GENTLEMEN,

I Return you hearty Thanks for the Honour you have done me, by the Favour of your Names to this little Treatiſe: Let me crave your further Aſſiſtance in noting down thoſe Errors that you may occaſionally find, and you will ſtill further oblige,

GENTLEMEN,

Your very humble Servant,

Sept. 24, 1767.

Daniel Fenning.

N. B. Gentlemen are taught Algebra, and the Uſe of the Globes, at their own Houſes, by the Author.

THE

THE INTRODUCTION.

CHAP. I.

DIALOGUE I.

Between PHILO, *a Tutor, or Master*; *and* TYRO, *a young Scholar*; *concerning the Rudiments of* ARITHMETIC.

TYRO *visits* PHILO.

Tyro. DEAR Sir, I am your very humble Servant —— You will pardon me, I hope—— I hear you have done instructing *Tyrunculus*, and I am come to lay claim to a former Promise, of your giving me a better Notion of common *Arithmetic*.

Philo. You please me very much, young *Tyro*, I assure you, to see you so willing to learn; but I hear you understand the first four Rules already.

Tyro.

Tyro. When I was at School I had ſome Knowledge of them, as I thought; but it was not well-grounded; and when I left School, inſtead of practiſing at Home, and making myſelf Maſter of what I learnt there, I bent my Mind to Play and Idleneſs, like other naughty Boys; and were it not for your kind Offer, I ſhould know the Want of it too late. I chooſe therefore, Sir, to begin at the very loweſt Branch, that I may ſee the Reaſon of what I am doing, and not learn by Rote, as too many School-boys do, to the great Diſcredit of their Maſter, Grief of their Parents, and their own future Ruin.

Philo. You ſay very right; for in beginning again you will be confirmed in what you know already; for I ſhall proceed with you the ſame as if you never had began, that others may be the better informed; therefore, for their Sakes, do not you be angry, if I ſhould dwell upon ſome Things longer than you may think there is Occaſion for, ſince I tell you the Reaſon before hand.

Tyro. Far be it from me, Sir, to take it amiſs; for though I do know ſomething of the firſt four Rules, I am ſenſible there are many Thouſands who know Nothing of the Matter, and you do well to conſider them alſo.

Philo. I am glad, *Tyro*, to ſee you ſo conſiderate; it gives me great Hopes of your being Maſter of what I am about to inſtruct you in.

B 1 A-

DIALOGUE II.

SECTION I.

Of Numeration, Addition *of* Whole Numbers, Money, Weights, *and* Measures, *&c.*

Tyro. WHAT is *Numeration,* and what does it teach ?

Philo. Numeration is the true Distinction and Pronunciation of Number; that is, it teaches us to write down, read, and express any Number or Numbers whatsoever: For the better understanding of which, observe the following Table.

N. B. The Letter C stands for an Hundred, and X for Ten.

TABLE.

	C of Millions.	X of Millions.	Millions.	C of Thousands.	X of Thousands.	Thousands.	Hundreds.	Tens.	Units.
Places	9	8	7	6	5	4	3	2	1
	9	8	7	6	5	4	3	2	1
		9	8	7	6	5	4	3	2
			9	8	7	6	5	4	3
				9	8	7	6	5	4
					9	8	7	6	5
						9	8	7	6
							9	8	7
								9	8
									9

Note I.

This Table you ought to get by Heart, at least, so as to understand the Nature of it.

Cast your Eye now *Tyro* upon it; you see that there are nine Places of Figures from Units

Units to Hundreds of Millions. All the Figures under the first Row are Units; those in the second Row towards the Left-Hand, are under the Place of Tens; all in the third Row are called Hundreds; those in the fourth Row Thousands, &c.

Now, in Order to know the Value of, or how to express any Number in the Table, I begin at the top Figure, towards the Right-Hand, and say Units (1); then Units, Tens, (Twenty-one, 21;) then Units, Tens, Hundreds, (three Hundred and Twenty-one, 321:) Thus I go on, and find the fourth Figure (*viz* 4.) under the Place or Name of Thousands, and accordingly, I call it four Thousand, which, joined to the other three Figures, will be four Thousand, three Hundred, and Twenty-one, 4,321. The fifth Figure being in the Place of Tens of Thousands, is thus read: Fifty-four Thousand, three Hundred, and Twenty-one, 54,321. Thus proceed till you come to the last Figure of all towards the Left-Hand, (which stands in the Place of Hundreds of Millions) and you will easily perceive, that those nine Figures are thus expressed: Nine Hundred, eighty-seven Millions, six Hundred, fifty-four Thousand, three Hundred, and Twenty-one, 9,87,654.3,21. Read this once more, and observe the Comma's, or Stops, that are put to the Figures; for they answer to, or correspond with the Stops in the Words that are written out at Length.

NOTE. 2.

You are further to observe, *Tyro*, that the *Numeration-Table* is not always set with these Figures just in the Form they here stand; for had they been any other nine Figures, they are numbered and expressed after the same Manner: For Instance, suppose they were 1,2,3,4,5,6,7,8,9, this is expressed after the same Manner, only instead of 987 Million, it is now 123 Million; instead of 654 Thousand, it is here 456 Thousand; and instead of 321, it is now 789. So also,

999,999,999

999,999,999 is read thus: Nine hundred, ninety-nine Million, nine hundred, ninety-nine Thousand, nine Hundred, and Ninety-nine, *&c.* So that you see it is only having a due Regard to the Places under which the Figures stand; for you may see that after the first three Places of Figures (*viz.* Units, Tens, and Hundreds) the next three Places have the Name of Thousands, the next three the Place of Millions, as you may observe by the following Table, which, I believe, will be of Service.

TABLE 2.

Millions Place.			Thousands Place.			Hundreds.	Tens.	Units.
1	2	3.	4	5	6.	7	8	9
	1	2.	3	4	5.	6	7	8
		1.	2	3	4.	5	6	7
			1	2	3.	4	5	6
				1	2.	3	4	5*
					1.	2	3	4
						1	2	3
							1	2
								1

Now here you see they are divided by Stops, or Periods, which is certainly a Help to the expressing the Numbers: Thus, suppose I would express the Number against which this Mark (*) is placed. I find the *Twelve* stands under the Place of Thousands, so that I say it is twelve Thousand, three Hundred and Forty-five, *&c.*

Tyro. I perceive it, Sir, very plainly: But, pray what Use are Cyphers of; or are they of no Signification?

Philo. Cyphers are of no Signification, when they stand alone, (thus ooo, or oooo, all stand for Nothing): Nor are they of any Signification placed before any Figure or Figures, thus, o2 is but 2; and ooo5 is but 5 still: But when

when Cyphers are put after Figures, it makes the Number ten, twenty, thirty, a hundred, or a thousand Times more in Value. Thus 1, by adding a Cypher, is (10) Ten, add two Cyphers, thus, 100, it is an Hundred, &c. as by the following Table.

TABLE 3.

1	One.
10	Ten.
100	An Hundred.
1000	A Thousand.
10000	Ten Thousand.
100000	An hundred Thousand.
1000000	A Million.
10000000	Ten Million.
100000000	An hundred Million.
1000000000	A Thousand Million.
10000000000	Ten Thousand Million.
100000000000	An hundred thousand Million.
1000000000000	Million of Millions.

And thus you may go on as far as you please.

Tyro I perceive it; but who could tell the Value, or express so many Figures out of the Course of the Table?

Philo. You talk like a Learner indeed. You see the twelfth Figure, by the above Table, has the Name of an hundred thousand Millions; therefore, were there 3, 4, or 5 Times, or 5 hundred Times as many Figures, it would be as easy to number them in Order, as well as if there were but 5 in all.

Tyro. That is a little surprising to me, I confess; for some think they do great Things, to number 9 or 10 Figures only.

Philo. Well, *Tyro*, we will not call that Pride altogether; for Pride is good in Learners, so far as it tends to Emulation only; that is, an earnest Desire to excel in Learning: But, pray observe, Suppose I had ever so many

many Figures to number; you plainly see, by the last Table, that the seventh Figure is Millions Place, and that the thirteenth Place has the Name of Millions of Millions; so also the nineteenth Figure would be Millions of Millions of Millions: But as the Word Millions would be repeated so often in a very large Number of Figures, as to render it tiresome, there is a shorter and much easier Way of expressing the Number by certain Words, which answer to every seventh Figure, and to any Degree of Millions, as appears by the following Table.

TABLE 4.

Dots.	
1	The 7th Figure from the Units Place is Millions;
2	13 is Bimillions, or Millions of Millions.
3	19 is Trimillions, or the 3d Deg. of Millions.
4	25 is Quartrillions, or the 4th Deg. of Millions.
5	31 is Quintillions, or the 5th Deg. of Millions.
6	37 is Sexquillions, or the 6th Deg. of Millions.
7	43 is Septillions, or the 7th Deg. of Millions.
8	49 is Octillions, or the 8th Deg. of Millions.
9	55 is Nonmillions, or the 9th Deg. of Millions, or Millions 9 Times repeated, *&c.*

Suppose it were required to number the 57 following Figures, 321, 987, 654, 321, 987, 654, 321, 987, 954, 321, 987, 654, 321, 987, 654, 321, 987, 654, 321.

Here I make a Period or Dot over every 7th Figure, and find there are 9 Dots in all; Then searching in the above Table, I find the 9th Dot to bear the Name of Nonmillions, the 8th of Octillions, *&c.* Therefore, I number the above Figures thus; three Hundred 21 Nonmillions, 987 Thousand; 654 Octillions, 321 Thousand; 987 Septillions, 654 Thousand; 321 Sexquillions, 987 Thousand; 654 Quintillions, 321 Thousand; 987 Quartrillions,

Quartrillions, 654 Thousand; 321 Trimillions, 987 Thousand; 654 Bimillions, 321 Thousand; 987 Million, 654 Thousand, 321. And thus you may go on to as many more Places, &c. Or you may number them by the second Part of the Table, and that is, by putting small Figures where the Dots stand, as 1, 2, 3, 4, till you come to 9: Then begin, and say $\overset{9}{321}$ of the 9th Degree of Millions, 987 Thousand; $\overset{8}{654}$ of the 8th Degree of Millions, &c. till you come to the End: And thus, by putting small Figures over every seventh Figure, you may number to the 20th, 30th, or any higher Degree of Millions whatsoever.

Tyro. I understand you well; and though it is likely there may never be so many Figures in any Sum; yet, it is good to know how to number them when a Question is asked, though it be for Fancy's Sake only. But now, *Philo*, if you please, I will ask you a few Questions, which I am at a Loss to know.

Quest. 1. How is *eleven Thousand, eleven Hundred*, and *Eleven*, set down in 5 Figures?

Philo. I know this is a common Question, and it is easily done, if you consider; for this is the Answer 12,111, *viz.* twelve Thousand, one Hundred, and Eleven.

Proof by ADDITION.

$$\left.\begin{array}{lr} \text{Eleven Thousand is} & 11{,}000 \\ \text{Eleven Hundred is} & 1{,}100 \\ \text{Eleven is} & 11 \end{array}\right\}\text{add}$$

Add 12,111, as above.

Quest. 2. How is *fifteen Thousand, fifteen Hundred*, and *Fifteen*, set down in five Figures? *Answer* 16,515.

But you are to observe, *Tyro*, that though these Oddities are easily answered you see; yet, I would not have you be concerned with them; because they are very badly

badly expressed, and are an abuse of language: for suppose you stood indebted to me, and should ask me what it was, and I should say fifteen pounds, fifteen shillings, and fifteen pence, would you not think it much better to say fifteen pounds, sixteen shillings, and three pence? Take this then for a rule, *Tyro*, that that is the best method of expression; that is the shortest, freest, and most natural. Therefore, tho' the aforesaid numbers run a little smoother in words than what I have now mentioned, (the reason of which is, because they are each but one whole number) yet is the sense of expressing fifteen thousand, fifteen hundred, and fifteen, very little better than fifteen pounds, fifteen shillings, and fifteen pence.

☞ Now there are several things a little dark in expression, relating to numbers, which are actually of use when they are known. I imagine, *Tyro*, you now and then read good books: You remember we read in the book of Kings, and in Isaiah, that an Angel destroyed in the camp of the Assyrians, an hundred, and fourscore and five thousand. How do you set this down properly?

Tyro. Why really, Sir, I am at a loss; and I have seen older persons than myself puzzled at it.

Philo. So have I; but it must be for want of considering. Is not fourscore and five the same as 85. Therefore, the number is one hundred eighty-five thousand, thus, 185,000.

Again, you read in the Revelations, of a multitude of ten thousand times ten thousand; which is 100,000000, viz one hundred million: and a little farther, you read of another, consisting of two hundred thousand thousand, which is, 200,000000 million, viz. two hundred million, which you will see demonstrated, Example the 3d and 4th in compendiums of Multiplication.

I shall finish Numeration with some of the old Roman numbers, as it may be of service.

A TABLE

A TABLE of Old ROMAN Numbers.

L. ſignifies, or ſtands for (50) Fifty.	
C. for (100) Hundred.	
CC. (200) Two Hundred.	
CCC. (300) Three Hnndred.	
CCCC. (400) Four Hundred.	
D. ſignifies (500) Five Hundred,	or thus IↃ.
DC. ſtands for (600) Six Hundred,	or thus IↃC.
DCC. (700) Seven Hundred,	or thus IↃCC.
DCCC. (800) Eight Hundred,	or thus IↃCCC.
DCCCC. (900) Nine Hundred,	or thus IↃCCCC.
M. (1000) A Thouſand.	or thus CIↃ.
MM. (2000) Two Thouſand, &c.	

ROMAN Numbers explained by FIGURES.

DL. 550.
DCCX. 710.
MDXIV. 1514.
MDCLI. 1651.
MDCCIX. 1709.
MDCCLXVII. 1767, &c.

And now, *Tyro*, we will proceed to Addition.

SECTION. II.

Of ADDITION.

Tyro. WHAT do you mean by Addition?

Philo. Addition ſignifies the gathering, collecting, and adding together, two, or more numbers into one ſum.

Tyro. How many parts are there in Addition?

Philo. Two, ſimple and compound.

Tyro. What is ſimple Addition?

Philo.

Philo. Simple Addition is that which consists of one simple or single name; that is to say, of whole numbers only, as pounds sterling, tons, yards, ells, or ounces, &c. For ten is ten, and a thousand is a thousand, in any number and quantity; tho' the quality or name be different; and they are all added by one Rule, namely by casting out the tens, and setting down in every line what is over ten, as you will see by and by.

Tyro. What does Compound Addition consist of?

Philo. Of Money, Weights, and Measures, viz. Avoirdupois Weight, Troy Weight, Apothecaries, and Goldsmiths Weight, &c. Likewise, Dry Measure, Liquid Measure, Cloth Measure, Square Measure, Long Measure, Land Measure, and Time. Of these in their order.

SECTION III.

ADDITION of whole Numbers.

THE Rule to be got by heart is,

For every ten in the units place, or first row of figures, you must carry one to the tens place, or second row, and so proceed; because ten units make ten, ten tens an hundred, and ten hundred a thousand.

Add	2	3	2		9
	1	2	1		7
	1	1	4		8
	5	2	3	and	4 together.
	—	—	—		—
	9 *Ans.*	8 *Ans.*	10 *Ans.*		28 *Ans.*

Here I say 5 and 1 is 6, and 1 is 7 and 2 is 9; which I place under the numbers, and it is done: but to prove whether

whether it be right, I begin at top, and caſt it downwards and find it comes to the ſame.

Secondly, If it amounts to juſt ten and no more (as in Example 3) ſet it down as before; only ſet the cypher under the row, and ſet the figure of 1 out towards the left hand. So alſo, in Example 4, I find it amounts to 28, which I ſet under the row, viz. the units 8 under the row itſelf and the 2 towards the left hand in the place of tens.

Thirdly, When you have two or three rows of figures, then, according to the Rule, add up the firſt row, or units places, and obſerve how many tens it contains, and if it comes juſt to even tens, ſet a cypher underneath, and carry as many ones to the next row, as there were tens in it; that is, if it be 20, carry 2, if it be 30, carry 3, if 40, carry 4, and for 50, carry 5, &c. And the laſt row of all ſet down what it amounts to, as if it were one ſingle row only.

An EXAMPLE at large.

Add	87625	pounds
	94915	
	62194	
	76547	
	81965	
	92198	
and	71964	together
	567408	*Anſ.*

Now obſerve once for all. I begin, and ſay, 4 and 8 is 12, and 5 is 17, and 7 is 24, and 4 is 28, and 5 is 33, and 5 is 38; this is the amount of the firſt row, or units place, but I muſt not ſet down the whole 38, but ſee how many tens it contains, and find it to be 3 tens and 8 over; this 8 I ſet under the row, and carry 3 to the next row, or tens place, ſaying, 3 that I carry and

and 6 is 9, and 9 is 18, and 6 is 24, and 4 is 28, and 9 is 37, and 1 is 38, and 2 is 40: This being just 4 tens, I set down a cypher, and carry 4 for the 4 tens to the next row, or hundreds place, saying 4 and 9 is 13, *&c.* and I find the third row to be 44, which is 4 tens and 4 over; therefore I set down 4, and carry 4 to the fourth row, or place of thousands, and find that it amounts to 27, then I set down the odd 7, and carry 2 to the 5th or last row, and find that it amounts to 56, and because this is the last row I set down the whole 56, that is, the 6 under the row, and the 5 to the left-hand.

Fourthly, When there are several numbers to be added together, consisting of some fewer and some of more figures, they are added after the same manner as before, only observe, in setting down the numbers on your slate, or in a book, that you be careful to set units under units, tens under tens, or else you will be puzzled in casting them up.

Ex. 6. Suppose I were to add 3417, 26, 184, 9, 271, and 3 together: I set them down as follows, which will be a standing rule for any thing of the like nature.

EXAMPLE 6.

Loads.
3417
26
184
9
271
3
3910

Here you see I set units under units, and tens under tens, and then I cast up from row to row as before directed.

Tyro. I underſtand it very well; but I ſhould have ſet it down thus, with cyphers, to ſupply the vacant places:

3417
0026
0184
0009
0271
0003
——
3910

Philo. It is quite ſuperfluous; for you have been told already, that cyphers before figures do not at all increaſe the value; beſides, it is neither ſo ſightly, nor advantageous to caſt up, nor are they ſeldom or ever uſed by accomptants, even in addition of money, to fill up any line or ſpace whatſoever, as you will ſee in *example* 13, of *Addition of money*,

More examples in whole numbers.

Buſhels.	*Ells.*	*Yards.*
471756	6715	65432
434176	46	2145
621985	2	234
942176	2176	67
354219	200	8
471625	6	76
982196	10	354
853294	1756	3256
	1	76258
5131427	10919	147830

And

And now, *Tyro*, I will set you two or three Questions by way of exercise.

QUESTIONS *to exercise* ADDITION *of whole numbers*.

Quest. 1. From *London* to *Rumford* is 11 miles, from *Rumford* to *Brentwood* 6, from *Brentwood* to *Chelmsford* 11, *from Chelmsford* to *Witham* 8, from *Witham* to *Kelvedon* 4, from *Kelvedon* to *Colchester* 10, from *Colchester* to *Manningtree* 9, and from *Manningtree* to *Harwich* 12: how many miles then is it from *London* to *Harwich*? *Answer* 71.

Tyro. I apprehend the question; it is only setting down the numbers as they stand in their order, as under.

	Miles,
From *London* to *Rumford*	11
From *Rumford* to *Brentwood*	6
From *Brentwood* to *Chelmsford*	11
From *Chelmsford* to *Witham*	8
From *Witham* to *Kelvedon*	4
From *Kelvedon* to *Colchester*	10
From *Colchester* to *Manningtree*	9
From *Manningtree* to *Harwich*	12
From *London* to *Harwich* ..	71 *ans*.

Quest. 2. A farmer has 7 fields, containing the following acres, *viz*.

	Acres.	
In one field are	25	How many acres are there in all? *Ans*. 192.
In another	57	
In another	18	
In another	25	
In another	15	
In another	43	
In another	9	

Ans. 192 acres.

 Quest.

Quest. 3. A draper has 6 pieces of cloth as under.

		Yards.	
* N° 1	containing	87	How many yards are there in all? *Ans.* 210.
2	— —	45	
3	— —	17	
4	— —	8	
5	— —	32	
6	— —	21	

Ans. 210 yards.

Quest. 4. How many days are there in the year, or in the 12 calendar months? First, set them down as follows.

		Days.
1st,	*January* has —	31
	February —	28
	March — —	31
	April — —	30
	May — —	31
	June —	30
	July —	31
	August — —	31
	September —	30
	October —	31
	November —	30
	December — —	31

Note, Every fourth year is called *leap-year*, and has 366 days; *February* having then 29 days.

Ans. 365 days.

* *Note* That in marking any sort of goods N° stands for Number.

Quest.

Quest. 5. How far is it from *London* to *Carlisle* in *Cumberland*, when,

	Miles.
From *London* to *Newcastle* is	149
From *Newcastle* to *Preston*	62
From *Preston* to *Lancaster*	21
From *Lancaster* to *Penrith*	50
From *Penrith* to *Carlisle*	19

Ans. 301 miles.

Tyro. I underſtand what you have ſhewn me very well.

Philo. Then I ſhall only leave one queſtion more for you to try at your leiſure.

Add 47, 697, 5, 91707, 100000, 26300175, 500, 62, and 987654321 together? *Ans.* 1014147514.

To prove ADDITION.

Tyro. How do you prove *addition?*

Philo. Two ways. *Firſt*, When you have caſt up any ſum, as before directed, then begin at the top and caſt downwards inſtead of upwards; and if the figures come the ſame, no doubt but the work is right: beſides, in things which require care, you ſhould make a practice of caſting every line, firſt upwards and then downwards.

The *ſecond* way is very well for learners, but too tedious for buſineſs; but as it is cuſtomary in ſchools to teach it, I ſhall ſhew you the method, which is as follows.

Let us take *Queſt.* 5, which amounts to 301 Miles; and to prove whether this be right, I cut off the top line of figures by a ſtroke with the pen, thus——:

```
      149
      ———
       62
       21
       50
       19
      ———
      301 Anʃ.
      ———
      152 Add this to the top 149.
      ———
      301 Proof. See Addition of money.
```

Then I begin to caſt the ſum up again as I did at firſt, (except the figures that are cut off, or that ſtand above the line) and find it amounts to 12, which is 2, and I cary 1; then I proceed to the ſecond row, and find it makes 15, ſo that this amounts in all to 152, which I place under the 301. *Laſtly*, I add this middle line 152 to the top line 149, and find that they make 301, which proves the firſt work to be right. I ſhall give you a further reaſon, when you come to examples of *money*, why this way is not ſo fit for practice in buſineſs as caſting the ſum upward and downward. And now we will proceed to

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SECTION IV.

ADDITION *of* MONEY.

Tyro WHAT is neceſſary to the learning of *Addition of money*.

Philo. Theſe three things: *firſt*, the rule; then the characters, and *thirdly*, pence-tables; all which ſhould

ſhould be perfectly got by heart before you pretend to caſt up money.

1. *The* RULE.

For every 4 farthings carry 1 penny to the pence; for every 12 pence carry 1 ſhilling to the ſhillings, and for every 20 in the ſhillings carry one to the pounds, which are caſt up by tens, as in whole numbers.

2. *Of the common* CHARACTERS.

Note 1. £ ſtands for pounds, *S* ſtands for ſhillings. *D* ſtands for pence. Or thus, £. *s*. *d*. pounds, ſhillings, pence.

Note 2. A farthing is one-fourth part of a penny, and is thus ſet down $\frac{1}{4}$. An half-penny is one-half, and is thus ſet down $\frac{1}{2}$. Three farthings being three-fourths, is thus ſet down $\frac{3}{4}$.

Note 3. 1 quarter, 1 half, and 3 quarters are alſo ſet thus, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.

3. *Of* PENCE-TABLES.

TABLE 1.

Pence.		*s.*	*d.*
20	is	1	8
30		2	6
40		3	4
50		4	2
60		5	0
70		5	10
80		6	8
90		7	6
100		8	4
110		9	2
120		10	0

TABLE 2.

Pence.		*s.*
12	is	1
24		2
36		3
48		4
60		5
72		6
84		7
96		8
108		9
120		10

By the help of these two tables, the first increasing by tens, or every 10 pence; and the other by even shillings, or every 12 pence, you will soon cast up any small sums, by a due regard to the following examples.

Tyro. But I hope, Sir, you will give me an example in length, by explaining it in words; for that will be of more service to me than an hundred examples without explanation.

Example 1.

	£.	s.	d.	
Add	9	4	2	
	8	5	3	
and	7	9	6	together
	24	18	11	

Example 2

	£.	s.	d.	
add	41	3	8	
	62	9	7	
and	47	11	6	together
	151	4	9	

Example 3.

	£.	s.	d.	
Add	95	16	2	
	68	14	10	
and	49	12	11	together.
	*			

Philo. You seem to be timerous, and doubt your own abilities without occasion. Come, pray try at the first sum?

Tyro. I see plainly how you do *that*, because the pence all added together do not exceed 12, nor do the shillings exceed 20; therefore, I set the amount of them under the row to which they belong, and find the total to be twenty-four pounds, eighteen shillings, and eleven-pence.

* I leave *example* 3 undone for your practice, and will give you another table for the more easily casting up the shillings.

Philo.

Philo. The same is to be observed in *Ex.* 2. Thus I say, 6 and 7 is 13, and 8 is 21 pence. Now, by the 1st *table*, 20 pence is 1*s.* 8*d.* therefore, 21 pence must be 1*s.* 9*d.* Or by the 2d *table*, I ask how many times 12 I can have in 21*d.* and find it 1, and 9 over; therefore, I set the odd 9 down under the place of pence, and carry 1 to the shillings, saying 1 that I carry and 11 is 12, and 9 is 21, and 3 is 24 shillings. Now, as 20*s.* make a pound, 24*s.* is 1*l.* 4*s.* therefore, I set the 4 under the shillings, and carry one to the first row of the pounds, casting them up as in *Addition of whole numbers*, and find it 151 *l.* so is the total 151*l.* 4*s.* 9*d.*

TABLE 3.

Shillings.		Pounds.	*s.*
20	is	1 -	
30		1 -	10
40		2 -	
50		2 -	10
60		3 -	
70		3 -	10
80		4 -	
90		4 -	10
100		5 -	

Please to add up the following sums.

Example 4.	Example 5.	Example 6.
£. s. d.	£. s. d.	£. s. d.
45 - 9 - 9	249 - 3 - 10	652 - 5 - 8
94 - 5 - 8	176 - 5 - 8	652 - 5 - 8
76 - 7 - 10	649 - 8 - 6	652 - 5 - 8
51 - 8 - 9	148 - 7 - 5	652 - 5 - 8
25 - 7 - 6	219 - 5 - 7	652 - 5 - 8
75 - 5 - 11	154 - 7 - 9	652 - 5 - 8
368 - 5 - 5		

Tyro. I begin at the row of pence, and find that it amounts to 53. Now, according to my *firſt pence-table*, 50 pence is 4*s.* 2*d.* therefore, 53 pence is 4*s.* 5*d.* Or by *table* 2, I find how many 12 pences I can have in 53 pence and find that 4 ſhillings is 48 pence, therefore, 53*d.* muſt be 4*s.* 5*d.* which odd 5 pence I ſet down under the row of pence, and carry the 4 ſhillings to the row of ſhillings, and find it amounts to 45. Now, by *table* 3, I find 45*s.* to be 2*l.* 5*s.* which odd 5*s.* I put under the ſhilllings, and carry 2*l.* to the pounds, which I caſt up as in *whole numbers* (by tens) and find the *total*, or whole amount, to be 368*l.* 5*s.* 5*d.*

Philo. Very well; but now you have done it, pray let me hear you ſay what it comes to; for I have known many ſchool-boys not able to read what they write or caſt up.

Tyro. 'Tis very true, ſir, but I think it is three hundred and ſixty-eight pounds, five ſhillings and five pence.

Philo. Very right; and I make no doubt by your careful proceeding, but you will underſtand the firſt four rules of *arithmetic* in a ſhort time. I leave *example* 5 and 6 undone for you, to try at your leiſure, after the ſame way and manner: and now I will try you with an example or two with farthings.

EXAMPLE. 7.			EXAMPLE 8.			EXAMPLE. 9.		
£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*
47	8	4½	271	7	6¾	278	4	3
15	5	10	146	5	8¼	164	8	9½
17	4	8½	956	8	3	215	1	10
49	9	6¼	178	7	8¾	464	5	4¼
15	10	7	246	9	7¼	176	7	8½
14	7	9¾	101	6	5	291	3	5
97	3	5¾	247	8	3½	456	7	6¾
14	9	6	604	9	8¾	278	5	7
Anſ. 270	19	9¾						

Tyro.

Tyro. I can do theſe as well as the other. *Firſt*, I begin with the farthings, ſaying, 3 and 3 is 6, and 1 is 7, and 2 is 9, and 2 is 11 farthings, that is, 2*d.* and 3 farthings over, which 3 farthings I ſet under the row of farthings (thus $\frac{3}{4}$) and carry 2 pence to the pence, and find they amount to 57 pence, which is 4*s.* 9*d.* This odd 9 I ſet under the pence, and carry 4 to the ſhillings, and find them 59, that is, 2 pounds, and 19 over, which 19 I ſet under the ſhillings, and carry 2 to the pounds, I find them come to 270; ſo is the total or anſwer 270*l.* 19*s.* 9*d.* $\frac{3}{4}$.

Philo. Very well done indeed: proceed in the ſame manner, and you will find *Ex.* 8. to be 2752*l.* 3*s.* 3*d.* $\frac{1}{4}$.

Tyro. I am obliged to you, ſir, and now I ſhould be glad you would give me inſtructions how you manage *double figures* in the ſhillings.

Philo. I will ſhew you three or four different ways, and you may take that which appears moſt natural and eaſy.

4. *Of caſting up* double figures *in the ſhillings, ſuch as* 16. 18. 17. *or the like.*

EXAMPLE 10.			EXAMPLE 11.			EXAMPLE 12.		
£.	s.	d.	£.	s.	d.	£.	s.	d.
64	17	10$\frac{1}{4}$	756	17	8	4715	11	10$\frac{1}{2}$
47	14	11	175	16	11$\frac{1}{4}$	1762	14	5
67	18	10$\frac{3}{4}$	476	11	9$\frac{1}{2}$	6471	14	6$\frac{1}{2}$
55	17	8$\frac{1}{2}$	187	17	10	1754	15	11$\frac{1}{4}$
49	19	11	256	19	5$\frac{3}{4}$	6474	19	8
64	15	5	175	14	11$\frac{1}{4}$	1762	17	6
351	04	8$\frac{1}{2}$						

METHOD 1. I caſt the farthings up (firſt upwards and then downwards) and find them 6, I therefore ſet down $\frac{1}{2}$, and carry 1 to the pence, which are 56, which is

is 4*s.* 8*d.* that is, 8 and I carry 4 to the ſhillings; but as there are two rows of ſhillings, or *double figures*, I firſt of all caſt up that row, or thoſe figures that ſtand towards the right-hand, ſaying thus, 4 that I carry and 5 is 9, and 9 is 18, and 7 is 25, and 8 is 33, and 4 is 37, and 7 is 44 the firſt row; then I come back upon the ſecond row, from the top to the bottom, calling every figure of 1, ten; ſaying, 44 and 10 is 54, and 10 is 64, (and ſo on) 74, 84, 94, and 10 is 104 ſhillings, which is 5*l.* and 4 ſhillings over, that is 4, and I carry 5 to the pounds, which I find amount to 351. So that the total is 351 *l.* 4*s.* 8*d.* $\frac{1}{2}$.

Method 2. Some like *this* way beſt; they caſt up the firſt row of ſhillings, as before, which comes to 44, that is 2*l.* 4*s.* and ſet the 4 ſhillings under the ſaid row, and carry the 2*l.* to the next row of ſhillings, counting 1*l.* for every two figures in the ſecond row, (becauſe two tens make 20*s.* or 1*l.*) thus in the above example, there are ſix figures of 1 in the ſecond row, and counting two of them for 1*l.* then ſix will be 3*l.* which with the 2*l.* that belonged to the 44 in the firſt row, make 5*l.* to be carried to the pounds as before.

Method 3. Others make uſe of *dots:* that is, they make a dot at every 4 in the farthings, every 12 in the pence, every 20 in the ſhillings, and every 10 in the pounds, which method is very eaſy, and ſuitable to lads of a dull comprehenſion, and a bad memory; becauſe they are not able to carry large number of pence and ſhillings in their mind, but frequently carry falſe from figure to figure.

Note, There are different opinions concerning pointing, or dotting: Mr. *Dilworth* highly recommends it; but Mr. *Fiſher* calls it ſlovenly and unneceſſary. It is not my province io determine between theſe Gentlemen: But this I will ſay, that every maſter ſhould try all methods, and let the ſcholar uſe *that* which his moſt natural and eaſy to his capacity. He muſt be very dull who can't tell that 2 tens make 20, or that 13

twenties

twenties make 260: and to stop or point at so small numbers, as 3, 4, 5, 6, *&c.* ought (if possible) to be avoided; because to add up well, and learn to cast out quick, is a great step towards *Multiplication*, and it would look very odd to see a person (after having done a long sum in that rule) dot every ten in casting up the product. I think therefore, there is no occasion to point or dot at every 4, 10, 12, or at even twenties, when a little practice will make it easy: but indeed when you come to do by 13, 16, 19, 28, or the like, there dotting must unavoidably be used, as you will see in the first example of *Averdupoise Weight*, wrought at large.

N. B. *If you are obliged to dot, mind and make them very small; if you can see them yourself, that is sufficient: but the first method, in my opinion, is best, except it be a very long sum, such as the side of a large book, a* bill, *or a* parish rate, *or* duplicate, *and then the following method is the most certain and expeditious, and is very easy.*

METHOD 4. *Of making and casting up long bills*, PARISH RATES, *&c.*

Tyro. How is the general way of casting up very long bills, rates, *&c.*

Philo. The rule is this: *for every* 60 *in the pence carry* 5 *to the shillings*, because 60 pence make 5 shillings: and *for every* 60 *in the shillings carry* 3 *to the pounds*, because 60 shillings make 3 pounds; and *lastly, cast the pounds up by tens, as before directed.*

Tyro. Please to give me an example at large.

Philo. I will shew you first, how to make a *parish rate*, which may be of service, and if you can cast it up well, you may also cast up any long bill, by the same rule.

The

The form of a PARISH RATE, *&c.*

	£.	s	d.
Allen Anthony, *Esq;*	3	7	8 ¼
Andrews John	1	14	7 ½
Baker James, *Esq;*	15	16	8
Carter Thomas		17	11 ½
Darby Abraham	5	17	5
Fernell Abraham		19	8 ¾
Honner Joseph	7		9
Kirton John, *Esq;*	21	14	7 ¼
Longman Thomas	5		11 ½
Lumley Edward	3	17	6
Manning Thomas		15	10 ¼
Martin William, *Esq;*	11	11	8 ½
Martin Job	7	17	8
Nicholas Abraham	2	13	9
Norton Daniel, *Esq;*	17	10	4 ½
Parker Esther		8	11
Powell Judith		5	10
Randall Nicholas	1	17	6 ½
Robinson Abraham, *Esq;*	15	11	11 ½
Robinson John	7	10	
Robinson James	1		10
Ruggles Nathaniel	7	11	4
Rumley Thomas		15	½
Solly Thomas, *Esq;*	12	11	
Sorsby William		14	
Spratly David	8		11
Swallow Thomas	1	17	4
Thompson John		19	11 ¾
Twist William, *Esq;*	7	7	
Walker John	1	15	6
Wayland Edward, *Esq;*	8	14	9 ½
Wingate Samuel		15	7
Worley Abraham,	5	4	10
	£. 189	19	7 ¼

First,

First, I begin with the row of farthings (as in the former questions) and find them amount to 27, which is 6 pence, and 3 farthings over; which 3 I set under the farthings thus ($\frac{3}{4}$) and carry 6 to the pence.

Secondly, for the PENCE.

I say 6 that I carry and 10 is 16, and 7 is 23, and 9 is 32, and 6 is 38, and 11 is 49, and 4 is 53, and 11 is 64; this being 4 above 60, I make a small dot close to the 11, and carry the 4 that is over to the next figure, saying 4 that I carry and 4 is 8, and 10 is 18, and 11 is 29, and 6 is 35, and 10 is 45, and 11 is 56, and 4 is just 60; therefore, as there is nothing over to carry to the next figure, I say 9 and 8 is 17, and 8 is 25, and 10 is 35, and 6 is 41, and 11 is 52, and 7 is 59, and 9 is 68; that is 8 above 60, therefore I make a dot, and carry 8 to the next Figure, saying, 8 and 8 is 16, and 5 is 21, and 11 is 32, and 8 is 40, and 7 is 47, and 8 is 55: now 50 pence is 4*s.* 2*d.* therefore 55 pence is 4*s.* 7*d.* which odd 7 pence I set under the pence, and carry the 4 shillings to the dots, counting (as I said before) 5 shillings for every dot, which are three in number, that is, 15 shillings; and the 4 I carried to them is 19 shillings: this 19 I carry to the shillings, and contrary to the other methods, I now work cross-ways, taking the *double figures* as I go along, *viz.* the right-hand figure first, and then the left-hand one belonging to it, counting it for 10, as follows.

Thirdly, for the SHILLINGS.

I say 19 that I carry and 4 is 23, and 5 is 28, and the 1 on the left-hand of it, which is always counted for 10, is 38, and 4 is 42, and 10 on the left-hand belonging to it, is 52, and 5 is 57, and the 10 belonging is 67: here, according to the same rule, I make a dot, and carry the odd 7 to the next figure, as I did in the pence; saying, 7 that I carry and 7 is 14, and 9 is 23,

and

and 10 is 33, and 7 is 40, and 10 is 50, and 4 is 54, and 10 is 64; that is, dot, and I carry 4, to 11 is 15, and 5 is 20, and 10 joining it is 30, and 11 is 41, and 10 is 51, and 11 is 62, which is dot, and I carry 2 to the next figure, proceeding in the same manner, as before directed, till I come to the top of all, and find there are 39 shillings, besides the last dot, that is 1 £. 19*s.* therefore, I set the 19 under the shilllings, and carry the odd 1 £. to the dots (counting 3 £. for every dot, because 60 shillings is 3 £.) and find them 6, which is 18 £. and 1 I carried to them is 19 £. which I carry to the first row of the pounds, casting them up by tens, as in *whole numbers*, and find them 189 £. So that the total of the rate, or bill, is 189 £. 19 *s.* 7 *d.* ¾. Pray, *Tyro*, run it over once more, and the method also, and you will not lose your labour.

Tyro. Sir, I see the nature of it very well; but why do you leave those vacancies in the shillings and pence, for most people fill them up with cyphers.

Phil. I know it is the common custom of schools, but it is a very idle one, nor is it so sightly or convenient; for the cyphers hinder sight, and prevent expedition in casting up. *See example 6, in whole numbers.*

5. *Of* CYPHERS, *where necessary, and where not.*

EXAMPLE 13, *without cyphers, according to the true Order of* BOOK-KEEPING.	EXAMPLE 13, *with cyphers, according to the common Custom of Schools.*
5478 - 9 - 6	5478 - 09 - 06
1 - 19 - 5	0001 - 19 - 05
- 6 - 8¾	0000 - 06 - 08¾
179 - -	0179 - 00 - 00
- 17 - ½	0000 - 17 - 00½
7 - 5 - 9	0007 - 05 - 09
1 - 17 - 7	0001 - 17 - 07
4 - ½	0000 - 04 - 00½
5670 - ¾	5670 - 00 - 00¾

Now,

Now, *Tyro*, I would aſk you which is moſt ſightly, *that* which ſtands clear, or the other confuſed with cyphers. And it is not only this, but every line of the ſum, and the total alſo, reads much better: for it is only five thouſand, ſix hundred, and ſeventy pounds, and three farthings; whereas the other, according to the manner of cyphers (and the too common cuſtom of ſchool-boys) muſt be thus read, *five thouſand, ſix hundred, and ſeventy pounds, no ſhillings, and no pence, three farthings.* And tho' cuſtom ſome years ago prevailed upon one of the moſt ingenious authors *, to uſe the *expreſſion* after this ſort; yet it does not at all juſtify the correctors of the laſt edition, ſince it is now quite ſuperfluous, and out of date.

Note, Though I ſay it be not cuſtomary to uſe cyphers, in *Addition of money*, yet, in *weights* and *meaſures* they are often made uſe of, and the learner may take his own fancy, that teaches himſelf; but it is my opinion that maſters in general would find it much eaſier and better, both to themſelves and ſcholars, to ſet their addition ſums quite clear, and uſe no cyphers at all before figures.

Tyro. I am obliged to you, ſir; and now, if you pleaſe, I would know how you prove *Addition*?

Philo. That is quite eaſy, *Tyro.*

6. *To prove* Addition *of* Money.

There are two ways, one by cutting off the top line (of which ſee under the proof of *Addition*,) but it is not practicable in buſineſs, the beſt way is this: begin at the bottom and caſt up to the top, noting down what it comes to; then begin at the top, and caſt the ſame row downwards, and if it amounts to the ſame as before, there is no doubt but the work is right, provided you obſerve carefully to ſet it down right.

* *Wingate*, Page 8.

SEC-

SECTION V.

Containing some farther explanation of things necessary to be known, with the manner of drawing out bills, *writing* notes, *and* receipts, *&c. being very proper to exercise the young beginner.*

Tyro. I Shall be at a loss under this section.

Philo. You are mistaken; for I shall not leave you to yourself wholly, but will explain every thing I am capable of, to your understanding; because I am sensible many school-boys can add up sums that are set them very well; but when they are called upon out of school, to write out, and cast up a small bill, or the like, how awkward do they go about it, and will set puzzling over it so long, that one would think they knew nothing at all of the matter: and when it is done, it is often set in such confusion, that it will puzzle much better scholars than themselves to cast it up; because they do not observe to place their figures right under one another, which remember, *Tyro*, you be always careful of, for it is a great advantage, and the work will look neat, and you will have the praise.

Tyro. 'Tis very true, sir, I have known several that could cast up a bill if others would figure it down, but have not had any notion of drawing it out themselves: nor can I say I have, and therefore should be obliged to you to shew me.

Philo. That I will; but first of all, it will be proper to shew you some *signs* and *contractions* made use of both in arithmetic and in business.

1. *Of contracted signs and characters used in common arithmetic.*

Note 1. This character (+) signifies addition, and when placed betwixt numbers shews that they are all to be added together.—Thus 5 + 7 × 11 + 9, is thus read 5 more 7, more 11, more 9, &c.

2. This

2. This character (—) signifies less and is the sign of subtraction, and shews the number after it, is to be taken out of the number before it: thus 11—7 is 11 less 7 which is 4; and 148—93 shews that 93 is to be subtracted from 48, &c.

3. This (×) is the sign of multiplication and signifies into, and shews that all the numbers betwixt which it is placed are to be multiplied continually into each other.—Thus, 4 × 5 × 3, shews that 4 is to be multiplied into or by 5 and that product by 3.

4. This (÷) is the sign of division and shews the number before it is to be divided by the number after it.—Thus 48÷6 shews that 48 is to be divided by 6.

5. This (:) signifies to.

6. This (::) signifies so is.

7. The 2 last characters put together thus (: :: :) are the sign of the rule of 3 or proportion. Suppose you see 2 : 4 :: 8 : 16; it is thus read, as 2 to 4, so is 8 to 16, &c. As *a* : *b* :: *c* : d. As *a* to *b* so is *c* to *d*, &c

8. This (=) is the sign of equality, and signifies that the figures or quantities placed before it are *equal to* those after it: thus 4 × 8 = 16, is read 4 more *equal to* 16. So also 25 — 16 = 9, is 25 less 16, or 16 subtracted from 25 *equal to* 9. So also 5 × 9 + 12 — 2 ÷ 5 = 11, is thus read; 5 *into* 9, *more* 12, *less* 2, divided by 5, is equal to 11.

9. This (√) is the sign of the square root.

10. This $\sqrt[3]{}$ the sign of the cube root.

2. *Of contracted words, and what they signify.*

Note 1. *Bt* stands for, or signifies *bought*.

2. *Dr*. is *debtor*. *Cr*. is *creditor*.

3. *Do*. stands for *ditto*, and signifies the same thing, place, or sort of goods, as was wrote in the line before; it is used by merchants, tradesmen, and accomptants, both in books and bills, to avoid writing the same thing over and over again.

4. C°

4. *Co.* signifies *company*; that is, when two or more are in trade and in partnership.

5. *Messrs.* signifies *Messieurs*, or gentlemen concerned together in some trade or business, and is used with the foregoing contraction, (*Co.*) on the head or front of the bills, instead of writing all their names at length: thus, suppose *John Sharp* to have a bill upon *Aaron Nelson*, *John Long*, and *Joseph Truman*, it would be ridiculous to write thus:

Mr. *Aaron Nelson*, Mr. *John Long*, and Mr. *Joseph Truman*, bought of *John Sharp*.—— But we write thus:

Messrs. *Aaron Nelson*, and *Co*. bt. of *John Sharp*, as you will see in the following example.

A Linen-Draper's BILL.

EXAMPLE I.

Messrs. *Aaron Nelson*, and *Co*. bt. of *John Sharp*.

1767.			£.	s	d.
Jan.	4	To 11 pieces of *Irish*, at 3*l*. *per* piece. —— ——	33	-	
	12	To 2 ditto, at do. ———	6	-	
Feb.	8	To long lawn, as by agreement	3	15	6
Apr.	12	To 2 pieces of check, as by do.	1	7	10
	28	To 4 do. as by do. ——	2	2	-
May	24	To 2 yards of dowlas, at 11 *d*.		1	10
Aug.	9	To a parcel, as *per* bill ———	1	15	6
		This bill I cast up as a common sum in *addition of money*, and find the total	48	2	8

Ex-

Example 2.

A Stationer's Bill.

Mr. *Samuel Long* bt. of *John Page.*

1767.		£.	s.	d.
May 19	2 ream of fool's cap ſuperfine paper ——— ———	1 -	10 -	
June 12	4 ditto ——— ———	3 -	-	
	20 ream of coarſe brown ———	1 -	10 -	6
Aug. 14	To ſundry goods, as *per* bill	3 -	19 -	6
	£.	10 -	-	

Note, Some perſons never uſe the word *To* at the beginning of the bill. *Note* farther, the word *To* is uſed in books of accompts on the *Dr.* or left-hand ſide. The word *By* is uſed on the *Cr.* or right-hand ſide.

And now, *Tyro*, I ſhall draw you out a few more bills, and leave you to ſet them on a ſlate, and caſt them up yourſelf, which will be of ſervice, by way of exerciſe.

Example 3.

A Goldſmith's Bill.

Mr. *James Proud* bt. of *Paul Fineſhew.*

1767.				
May 9	To 1 Diamond ring ——— ———	15 -	-	-
	2 Pair of ſilver ſalts ———	2 -	2 -	6
	1 Quart tankard ——— ———	10 -	10	
	1 Pint do. ——— ———	4 -	7 -	6
	1 Dozen of knives and forks —	3 -	2 -	6
	1 Silver tea kettle ——— ———	5 -	5 -	
	12 Tea-ſpoons, tongs, and cream-pot	1 -	15 -	
	Total £.			

EXAMPLE 4

A Tailor's BILL.

Mr. *Robert Patience* Dr. to *John Trimmer*.

1767		£.	s.	d.
Mar. 15	To 2 yards of cloth, at 18 *s. per* yard	1	16	-
	Making your coat	-	8	-
	4 yards and a half of shalloon, at 2 *s.*	-	9	-
	Buckram, staytape, and canvas	-	3	6
	Silk, twist, and mohair	-	2	8
	Making a suit for your son	-	15	9
	Buckram, silk, twist, mohair, *&c.*	-	4	3
	Buttons to the same	-	3	6
		£.		

Now, from these four examples, '*Tyro*, you may form any other tradesman's bill in due order; and as for making out or balancing any reckoning, between one person and another, to see how much is due to either, is not the work of *addition*; but I shall explain it very fully to you in *subtraction*.

2. *The manner, or common form of* receipts, *and* notes of hand.

Tyro. How am I to give, or write a *receipt?*

Philo. According to what money you receive, and the persons you receive it of, or for. Let us take the stationer's bill in example 2, which is 10 *l.* Now, if he receives it in full, the receipt will run thus:'

August 6, 1767. *Received of Mr.* Samuel Long *ten pounds, in full.*

John Page.

£. 10.

If

If the receipt be wrote upon the bill itself then this is better:

Received at the same time the contents in full.

John Page.

EXAMPLE 2. If only part be received, then thus:

August 16, 1767. *Received of Mr.* Samuel Long *five pounds, on accompt.*

John Page.

£. 5

EXAMPLE 3. If the person that receives the money be a son or servant, he must write thus:

August 16, 1767. *Received of Mr.* Samuel Long *ten pounds in full, for my father (or master.)*

John Page, *jun.*

EXAMPLE 4. For rent.

June 14, 1767. *Received of Mr.* John Lumley *twelve pounds, ten shilllings, for half a year's rent, due at* Lady-Day *last.*

Abraham Gripe.

£. 12 - 10

EXAMPLE 5. When there has been an account of long standing, and at last the two parties agree to have a reckoning, but still he that owes money upon the balance has none at that time to pay, then the following form is counted better than a common note of hand, because it shews the reason of such an acknowledgment: but then this writing should be drawn in the book of the person to whom the balance is due, and if signed in the presence of witnesses the better. The form is thus:

August

August 16, 1767. *Reckoned and balanced all accompts, and I* Samuel Long *acknowledge myself to be indebted to* John Page *three pounds, ten shillings, which I promise to pay on demand, for value received. Witness my hand,*

Test. Samuel Long.
Abraham Justice.

EXAMPLE 6. When a person has no money about him, or has his money in other persons hands, and gives you a note or draught upon them for the payment of any sum, it is wrote after this manner:

Sir, please to pay to Mr. John Page, *or bearer, three pounds, ten shillings, and place it to the accompt of*
Your humble servant,
To Mr. Jonathan Trusty. Samuel Long.

3. *Of the Value of the common coins used in* England, *how they are expressed, and how set down.*

A *Port,* or *Portugal*-piece, is set down 1 *l.* 16 *s.* but is expressed, or commonly called a six and thirty.
A double *Port* is 3 *l.* 12 *s.*
A *Moidore* is set down 1 *l.* 7 *s.* but called a seven and twenty.
A *Guinea* is 1 *l.* 1 *s.* and expressed a guinea.
A *Crown* is expressed a crown; but set down 5 *s.*
Half a crown is 2 *s.* 6 *d.*
A *Tester* is six-pence.
A *Groat* is four-pence.

An useful EXAMPLE.

A servant laid out cash as follows: for *coals* a guinea and a half. *Cloth* three and twenty and six-pence. *Meat* seven groats. *Butter, eggs,* and *bacon* nineteen-pence half-penny, and *thread* seven farthings. What was laid out in all?

Tyro.

Tyro. I ſet them down as follows.

	£.	s.	d.
For Coals	1	11	6
Cloth	1	3	6
Meat		2	4
Butter, eggs, and bacon		1	7½
Thread			1¾
In all	2	19	¼

Philo. Very well done indeed. *Obſerve further then*, that from *one* to *two* ſhillings, and from *one* or *two* pounds, the *expreſſion* is different from the *ſetting-down*. Thus, 1 *s.* 10 *d.* ½ is expreſſed two and twenty-pence halfpenny; and 1 *l.* 19 *s.* 6 *d.* is expreſſed nine and thirty and ſix-pence.

So alſo, though theſe numbers, 1300, 1753, and 2500, are properly, one thouſand three hundred, one thouſand, ſeven hundred, and fifty three, and two thouſand, five hundred; yet they are thus expreſſed; thirteen hundred, ſeventeen hundred and fifty-three, and five and twenty hundred.

Tyro. I heartily thank you, kind ſir, and if it were not too troubleſome, I could wiſh you would ſet me a few queſtions by way of exerciſe.

Philo. You do very well to aſk me, *Tyro*; but it will be better at the end of *Addition*, where I ſhall give you ſome uſeful examples.

SECTION V.

AVOIRDUPOIS WEIGHT.

Tyro. PRAY what is the uſe of this rule, or what is weighed by it?

Philo. Moſt things that are commonly dealt in, ſuch as *grocery-wares*, and alſo, *cheeſe*, *butter*, *ſoap*,

candles, *allum*, *brass*, *iron*, *copper*, *salt*, *hemp*, and all such sort of goods.

Tyro. What are the different names or denominations of the weights used in this rule.

Philo. The *greatest* denomination is a *ton*, and the *least* a *dram*. They run thus in order: *tons*, *hundreds*, *quarters*, *pounds*, *ounces*, and *drams*; of which is composed the following table, with the characters that stand for each denomination after them.

The TABLE.

16 Drams	make	1 Ounce marked thus,		*oz.*
16 Ounces		1 Pound		*lb.*
28 Pounds		1 Quarter of an hundred		*qr.*
4 Quarters		1 Hundred weight		*Ct.*
20 Hundreds		1 Ton		*T.*

Tyro. Should I get this table by heart?

Philo. You may do as you please: I know some masters call it unnecessary; but if you take my advice, learn every table perfectly. 'Tis true, you may do the sums without it, if you have the table before you, but it would be a very bad thing to make an excuse, by saying, you could do such a sum if you knew the rule by heart; and I have known many a lad lament the omission.

Tyro. I don't doubt it at all, and I will take your advice: please to tell me how you cast up those sums.

Philo. The same as in *addition of money*, only you stop (as is plain by the table) by different figures; and for your further information, I shall put over every row and denomination, the quantity you are to stop at, or do by, and shall give you one example at large as a standing rule for all that follows.

Ex-

EXAMPLE 1.				EXAMPLE 2.			
(10)	(20)	(4)	(28)	(10)	(4)	(28)	(16)
Tons	*C.*	*qrs.*	*lb.*	*C.*	*qrs.*	*lb.*	*oz.*
25 -	14 -	1 -	15.	42 -	1 -	17 -	10
18 -	11 -	2 -	16	17 -	2 -	19 -	14
27 -	17 -	3 -	21.	21 -	3 -	22 -	11
46 -	14 -	1 -	17.	17 -	2 -	17 -	12
16 -	17 -	2 -	15	24 -	1 -	22 -	15
45 -	15 -	1 -	17.	65 -	3 -	17 -	13
16 -	17 -	3 -	14	24 -	1 -	21 -	10
198 -	9 -	1 -	3				

I begin at the *lbs.* and ſay, 14 and 7 is 21 and the 1 on the left-hand (which is always called 10) is 31; that is, 3 above 28; therefore, I make a dot againſt the 7, and carry the odd 3 forward; ſaying, 3 that I carry and 5 is 8, and 10 on the left of it is 18, and 7 is 25, and 10 on the left is 35, that is, 7 above 28; then I dot again, and carry 7 to the next figure, ſaying 7 and 1 is 8, and the 2 on the left-hand, which ſtands for 20, is 28; therefore, as there is nothing over, I only ſay 16 and 5 is 21, and 10 is 31, that is, 3 above 28, which 3 I place under the row of pounds, and then telling my dots, I find them 4 (that is 4 qrs.) which I carry to the next row of qrs. ſaying, 4 and 3 is 7, and 1 is 8, and 2 is 10, and 1 is 11, and 3 is 14, and 2 is 16, and 1 is 17 quarters. Now as 4 quarters make an hundred, I aſk how many *fours* I can have in 17, and find 4 *fours*, and 1 over, that is, 4 hundred, 1 quarter, which 1 I place under the row of quarters, and carry the 4 to the hundreds, which I caſt up by *twenty*, the ſame as in *addition of money*, ſaying, 4 that I carry and 7 is 11, and 5 is 16, and 7 is 23, and 4 is 27, and 7 is 34, and 1 is 35, and 4 is 39; then I come back with the tens, ſaying, 39, and 10 is 49, and 10 is 59, and ſo on, 69, 79, 89, 99, and 10 is 109, which is 5 twenties, and

9 over; that is, 5 tons, and 9 hundred over, which 9 I place under the hundreds, and carry 5 to the first row of the tons, and casting them up as whole numbers by tens, I find the first row 48, that is, 8, and I carry 4 to the next row, which amounts to 19. Therefore the total is 198 *tons*, 9 *c.* 1 *qr.* 3 *lb.*

☞ This is a standing rule, *Tyro*, for all your other sums in *Addition*, which are cast up after the very same manner: those that you find done are for your information and satisfaction; and such as are left undone are for your practice. Are you satisfied with what I have told you?

Tyro. Sir, I am; but in *example* 2, I perceive you set 10 over the *hundreds* place, contrary to the table; whereas in *example* 1, you have set 20 over the *hundreds* place, which I own puzzles me at present.

Philo. That is for want of a little consideration, *Tyro*; for whatever *name* or *denomination* stands first (that is, whatever you add up last) is always added up like *whole numbers*, by *tens*; be they *tons*, *hundreds*, *pounds*, *shillings*, *yards*, *ells*, or any thing else, as you will see hereafter.

Tyro. I am obliged to you, sir, and desire no further instruction in *Addition*, but only the rules and examples to go by.

Philo. You shall not want for either, and they are all done after the same manner as this example before you, though by different figures: and pray take *notice*, all those examples that you find ready done, are not to indulge you in idleness; but the answers are inserted for your information and satisfaction; and those that are left undone are for your exercise, practice, and improvement; and though I told you in *addition of money* it is not customary to use cyphers; yet, if you chuse it, take your own way; but you will find it better to leave them out, when there is no occasion to use them.

AVOIR-

AVOIRDUPOIS *small weight.*

Tyro. What is the uſe of this rule?
Philo. It is chiefly uſed by ſuch as deal in ſilk, worſted, thread, &c. by *retail*, or in ſmall quantities only; that is, from a dram to pounds, 16 drams make an ounce, and 16 ounces 1 pound.

EXAMPLE 1.

(10) *lb.*	(16) *oz.*	(16) *d.*
2	10	5
4	7	3
1	6	11
6	9	0
3	7	9
18	8	12

EXAMPLE 2.

(10) *lb.*	(16) *oz.*	(4) *qrs.*
3	14	2
1	11	1
3	9	0
2	01	2
2	00	3

Note, *Worſted* is weighed by ounces and quarters, as in *example* 2, and no drams are uſed here, nor in retailing many other commodities.

Of WOOL.

Wool is weighed by the *clove*, *ſtone*, *tod*, *wey*, &c. as follows.

7 Pounds make 1 clove
2 Cloves, or 14 *lb.* 1 ſtone
2 Stone, or 28 *lb.* 1 tod
6 ½ Tod 1 wey
2 Weys 1 ſack
12 Sacks 1 laſt.

Note, In ſome places 7 tod are allowed to 1 wey, and 12 ſcore, or 240 *lb.* is called a *pack of wool.*

Note farther.

A *Firkin* of Soap is 64 *lb.*
A *Firkin* of butter 56 *lb.*
A *Clove* of cheese 8 *lb.*
A *Wey* of cheese in *Essex* is 32 cloves, or 256 *lb.*
A *Wey* in some parts of *Suffolk* is the same; but in other parts of it, 42 cloves, or 336 *lb.* make a wey.

☞ *See* another table of *weights* and *measures, section* 6.

TROY-WEIGHT.

Tyro. Of what use is *troy-weight?*

Philo. By this is weighed gold, silver, jewels, electuaries, *&c.* and liquors in general.

TABLE.

Note,	24 Grains	make	1 Penny-wt.	*dw.* thus
	20 Penny-wts.		1 Ounce.	*oz.*
	12 Ounces		1 *lb. troy.*	*lb.*

Note 1. You need not *point* or *dot* at any row except the first, where you do by 24; the others are easy and common, as before, the penny-weights being cast up by 20, are done the same as shillings; and the ounces being done by 12, are cast up like pence, in *addition of money.*

10)

(10)	(12)	(20)	(24)	(10)	(12)	(20)	(24)
lb.	*oz.*	*dw.*	*gr.*	*lb.*	*oz.*	*dw.*	*gr.*
42 -	11 -	14 -	18.	647 -	11 -	17 -	16
26 -	9 -	10 -	21	197 -	10 -	15 -	23
49 -	10 -	11 -	11	494 -	11 -	19 -	14
65 -	8 -	14 -	12.	165 -	9 -	15 -	21
73 -	9 -	15 -	16	219 -	1 -	11 -	19
87 -	11 -	18 -	11	648 -	8 -	15 -	21
347 -	2 -	5 -	17				

Note 2. That 1 *lb. avoirdupois-weight* is equal to about 14 ounces 12 penny weights *troy*.

Note 3. Custom only introduced *avoirdupois-weight*, and beer-*measure*: for, according to the statute laws, there should be but one *weight* and one *measure* throughout the whole realm, as you may see under *dry-measure*, *Note* 1. Therefore, it is evident, that from these two different weights, came the different sorts of measures, as you will more plainly see under *dry-measure* and *liquid-measure*.

Note farther the value of gold *and* silver.

Gold.	£.	*s.*	*d.*
That 1 Pound weight of gold is worth	48 -	0 -	0
1 Ounce —— ——	4 -	0 -	0
1 Penny-weight —— —	0 -	4 -	0
1 Grain —— ——	0 -	0 -	2

Silver.	£.	*s.*	*d.*
That 1 Pound weight of silver is worth	3 -	0 -	0
1 Ounce —— ——	0 -	5 -	0
1 Penny-weight ——	0 -	0 -	3
1 Grain about ½ a farthing.			

APOTHECARIES WEIGHT.

Tyro. What is the use of this weight?

Philo. By it *apothecaries* mix and compound their medicines, their pound being the same as the pound *troy*, only

only differenely divided, as you see in the following *Table*.

Note, Though *apothecaries* mix their medicines by this rule, they buy and sell their drugs by *avoirdupois-weight*.

The TABLE.

20 Grains	make	1 Scruple	℈ thus marked.
3 Scruples		1 Dram	ʒ
8 Drams		1 Ounce	℥
12 Ounces		1 Pound	℔

(10) ℔	(12) ℥	(8) ʒ	(3) ℈	(20) gr.
42	10	7	2	19
17	8	5	2	11
43	7	6	1	10
15	4	5	1	9
47	11	3	1	8
64	9	6	2	15
232	5	4	0	12

(10) ʒ	(3) ℈	(20) gr.
18	1	14
12	2	17
41	1	10
24	2	15
16	1	11
17	2	15

DRY MEASURE.

Tyro. What is the use of *dry measure*?

Philo. By this rule is measured all dry goods, such as coals, sand, salt, fruit, oysters, wheat, barley, peas, and other grain, as appears by the two following tables.

TABLE 1.

2 Pints	make	1 Quart
2 Quarts		1 Pottle
2 Pottles, or 8 pints		1 Gallon
2 Gallons, or 16 Pints		1 Peck
4 Pecks		1 Bushel
4 Bushels		1 Coomb
2 Coomb, or 8 bushels		1 Seam, or quarter
5 Quarters, or 40 bushels		1 Load

TABLE

TABLE 2. *For Coals.*

4 Pecks	make	1 Bushel
9 Bushels		1 Quarter of a Chaldron
4 Quarters, or 36 bushels		1 Chaldron
Note, 5 Pecks		1 Bushel, water-measure

Note 1. That 33 *cubic* inches, and 3 fifths, make a *corn* pint, 268 inches, and 4 fifths, a *corn* gallon, and 2150 inches, 2 fifths, a true *Winchester* bushel, according to act of parliament, made in 1697, which says, that every round bushel with an even bottom, that is, 18 inches wide, and 8 inches deep, shall be deemed a true legal *Winchester bushel.*

(10) (5) (8) (4) *Loads qrs. B. Pecks*	(10) (4) (9) *Chal. qrs. Bush.*	(10) (36) (4) *Chal. B. Pecks.*
24 - 4 - 7 - 3	64 - 2 - 8	47 - 31 - 2
19 - 3 - 5 - 2	17 - 1 - 6	19 - 17 - 1
49 - 2 - 3 - 1	48 - 2 - 3	47 - 16 - 2
47 - 3 - 1 - 2	17 - 1 - 4	56 - 14 - 1
56 - 1 - 5 - 3	96 - 2 - 5	17 - 13 - 2
17 - 3 - 4 - 2	17 - 3 - 7	47 - 27 - 3
215 - 4 - 4 - 1		

Note 2. The common received contents of a *corn* gallon is 272 inches. For *dry measure* is a mean, as it were, between *wine* and *beer* measure. For as 12 *oz. troy* is to 224 inches; so is 14 $\frac{1}{2}\frac{2}{0}$ *oz.* to 272 inches nearly. *See note* 3, in *wine measure.*

Note 3. That *salt*, *sea-coal*, and many other commodities, are heaped in the measuring in general; and where they are not, it is customary to allow 5 struck pecks to the bushel. *Bran* is double measure; that is, 2 pecks struck are allowed for one peck.

Observe further.

A Score of coals — — is	21 chaldron, that
is, those that buy 20 chaldron have	21 for 20
A sack of coals ———	3 Bushels
A sack of corn in common ——	4 Bushels
A sack of flour ———	5 Bushels
A load, called a market load —	5 Bushels
A load in general means ——	40 Bushels
A wey is 10 quarters, or 2 loads	80 Bushels
A last in some places is 12 wey, *viz.*	960 Bushels

See *section* 6, in *Addition*.

LIQUID-MEASURE.

Tyro. What is the use of this *measure?*

Philo. All sorts of wine, spirituous liquors, and beer and ale are measured by it, under the names of *wine-measure*, and *Winchester-measure*.

1. *Of* WINE-MEASURE.

TABLE.

2 Pints	make	1 Quart
4 Quarts	make	1 Gallon
10 Gallons	make	1 Anchor of brandy, or rum
18 Gallons	make	1 Runlet
31 $\frac{1}{2}$ Gallons	make	1 Barrel of wine or vinegar
42 Gallons	make	1 Tierce
63 Gallons	make	1 Hogshead
2 Hhds. or 126 gallons	make	1 Butt, or pipe
2 Pipes, or 252 gallons	make	1 Tun

Note 1. A *puncheon* is 8 anchors, or 80 gallons; but any cask between a *hogshead* and a *pipe*, is called a *puncheon*.

(10) Tuns.	(2) Pipes.	(2) Hhds.	(63) Gall.
47	1	1	21
49	0	1	57
64	1	0	16
45	1	1	18
27	0	0	15
56	1	1	17
991	1	0	18

(10) Hhds.	(63) Gall.	(8) Pints.
47	19	7
26	14	1
94	17	5
57	47	6
19	14	4
45	49	5

Note 2. *Cyder, perry, oil, vinegar,* &c. are bought and fold by this measure; and milk is alfo fold in the city of *London* by it; there being no ftandard to the contrary, corrupt cuftom has reduced the largeft of liquid pints to one half its proper quantity.

Note 3. The *wine-pint* (according to cuftom) is reckoned to contain 28 *cubic* inches, and 7 eighths; and the *gallon* 231 inches. But by an experiment made at *Guild-Hall,* in *London,* (1688) a veffel containing but 224 *cubic* inches, was filled with water, and carefully emptied into the *wine gallon* kept there, which did exactly fill it. But notwithftanding *this,* it was thought proper to continue 231 inches to the gallon, which remains to this day. And from this increafe of the *wine gallon* (anfwering to *troy-weight*) came the increafe of the *beer gallon,* which anfwers to *avoirdupois-weight.* See *note,* 1. in *Winchefter measure*

2. WINCHESTER-MEASURE.

Tyro. What are the different meafures and denominations for *beer* and *ale?*

Philo. They are as under

TABLE.

2 Pints	make	1 Quart
4 Quarts	make	1 Gallon
9 Gallons	make	1 Firkin
2 Firkins, or 18 gallons	make	1 Kilderkin
2 Kilderkins, or 36 gallons	make	1 Barrel
1 ½ Barrel, or 54 gallons	make	1 Hogfhead
2 Hhds, or 3 bar. or 108 gall.	make	1 Butt
2 Butts, or 216 gallons	make	1 Tun

Not-

Note 1. That 35 *cubic* inches 1 quarter make a *beer* pint, and 282 (nearly) 1 gallon, which answers to *avoirdupois weight*. For as 12, the ounces in 1*lb. troy*, is 231, the inches in a *wine* gallon; so is $14\frac{12}{20}$ ounces to 282, the inches in a customary beer gallon. See *note* 2, *Troy weight*.

(10) *Butts*	(2) *Hhds.*	($1\frac{1}{2}$) *Bar.*	(2) *Kild.*	(2) *Firk.*	(9) *Gall.*	(10) *Butts*	(3) *Bar.*	(36) *Gall.*
15	1	0	1	1	5	64	1	27
23	0	1	0	1	2	15	2	18
41	1	1	1	1	3	21	1	14
22	1	0	1	0	2	25	0	15
7	1	1	0	1	5	62	1	16
1	0	1	1	0	4	5	0	29
112	1	1	1	0	3			

Note 2. That in gauging *beer* and *ale* in *London*, 32 gallons is a barrel of *ale*, and 36 a gallon of *beer*; but in other places 34 gallons is a barrel, one with another, and *common brewers* in the country allow victuallers 36 gallons of both sorts to the barrel.

CLOTH-MEASURE.

Tyro. What are the *denominations* of this measure?

Philo. *Yards*, *quarters*, and *nails*: also, *ells English*, *ells Flemish*, and *French ells* as appears by the table.

TABLE.

2 inches and a quarter	make	1 nail
4 nails	make	1 quarter of a yard
4 quarters	make	1 yard
3 quarters of a yard	make	1 ell *Flemish measure*
5 quarters, or 1 yd. 1 qr.	make	1 ell *English*
6 quarters	make	1 *French* ell

Note 1. That things in common, such as *woollen*, *linen*, *silk*, *tape*, *cord*, &c. are measured by the yard: but *hollands* are in general measured by the ell *English*, and *tapestry* by the ell *Flemish*.

Note

Note 2. That 16 nails make a yard, 20 nails an ell *English*, and 12 nails an ell *Flemish*.

(10)	(4)	(4)	(10)	(5)	(4)	(10)	(3)	(4)
Yds.	*qrs.*	*Nails*	*Ells Eng*	*qrs.*	*N.*	*Ells Fl.*	*qrs.*	*N.*
47	3	2	64	4	3	17	2	3
19	2	1	17	2	1	41	1	2
16	3	2	19	3	2	19	2	1
14	1	1	15	4	3	64	1	2
17	1	2	16	2	1	14	2	1
21	2	1	17	2	3	25	1	3
137	2	1						

LONG-MEASURE.

Tyro. What does *long measure* teach?

Philo. To know the length or breadth of any thing, and the distance of one thing, or place from another, as by the following table.

TABLE.

3 barley corns	make	1 inch
12 inches	make	1 foot
3 feet	make	1 yard
2 yards, or 6 feet	make	1 fathom
$5\frac{1}{2}$ yards	make	1 rod, pole, or perch
40 rods	make	1 furlong
8 furlongs	make	1 mile
3 miles	make	1 league
20 leagues, or 60 miles	make	1 degree
360 degrees	make	the circumference of the earth and sea

Note 1. An hand, or hand's breath in horsemanship is 4 inches.
A common pace is 2 feet 6 inches.
A geometrical pace is 5 feet.

(10) Deg.	(20) Leag.	(3) Miles	(8) Fur.	(40) Rods
25	19	2	7	21.
17	14	1	5	19
16	8	2	2	15.
21	5	1	4	27
47	7	2	1	14.
15	10	1	5	37
144	7	0	3	13

(10) Rods	($5\frac{1}{2}$) Yds.	(3) Feet	(12) Inches.
42	4	1	11
61	3	0	10
17	1	1	9
24	0	2	8
16	2	1	7
72	1	0	6

Note 2 According to the table 60 miles make one degree, therefore, the earth is 21600 miles round: but $69\frac{1}{2}$ miles (very nearly) make but one degree, and therefore the circumference of the earth is about 25000 miles; as you will fee in *Reduction*.

LAND-MEASURE.

Tyro. What are the denominations of *land meaſure?*

Philo. Almoſt the ſame as in *long-meaſure*; but as they never regard the inches and barley-corns, this table is ſufficient.

TABLE.

$5\frac{1}{2}$ yards, or 16 feet $\frac{1}{2}$	make	1 rod, pole or perch
40 rods, or poles	make	1 furlong in length
40 rods in length, and 1 in breadth	make	1 rood, or quarter of an acre
4 roods, or quarters	make	1 acre

Note 1. That though $16\frac{1}{2}$ feet make a ſtatute pole or rod, yet it is cuſtomary in ſome low, fenny countries, and barren lands, to allow 18 feet, and in meaſuring foreſts 21 feet to the pole.

(10) Acres	(4) Roods	(40) Poles	($5\frac{1}{2}$) Yds.
7	1	14.	1
9	3	27	2
4	1	29.	3
15	2	27	4
37	1	18	$4\frac{1}{2}$

(10) Acres	(4) Roods	(40) Poles
15	1	19
26	2	24
19	3	27
47	1	15

Note

Note 2. The common inftrument ufed in meafuring land is an iron chain, containing 100 links, which is 4 rods, or 22 yards in length; therefore 10 chains in length, and 1 in breadth, make an acre, and 8 chains in length only make a mile, or 1760 yards.

SQUARE-MEASURE.

Tyro. What do you mean by *fquare meafure?*

Philo. You are not to expect, *Tyro*, as yet to know the nature of it; it is fufficient at prefent, that you only know this, that *long-meafure* fhews you only the length or breadth of any thing; but *fquare-meafure* tells you the content of any thing, which you can have no notion of till you have learnt *Divifion*; but it will not be amifs to learn the following table by heart (at your leifure) that you may be the better able to underftand it by and by.

16 fquare quarters	make	1 fquare inch
144 fquare inches	make	1 fquare foot
9 fquare feet	make	1 fquare yard
30 fquare yards. 1 qr. or 272 fquare feet 1 quarter	make	1 fquare rod, or pole
160 fquare rods	make	1 acre of ground

Note. An example or two will be fufficient.

(10) *Yards*	(9) *Feet*
45	6
29	3
47	8
25	5
148	4

(10) *Yards*	(9) *Feet*	(144) *Inches*	(16) *Qrs.*
17	5	71	3
64	2	14	6
16	4	17	12
78	5	94	11

Of TIME.

Tyro. What is *time*, and how is it divided?

Philo.

Philo. *Time* ſhews us the beginning, mutation, (or changing) continuation, and ending of all mutable things. It is meaſured by *years*, *months*, *days*, *hours*, *minutes*, and *ſeconds*, and divided as follows, which will ſerve all common purpoſes.

TABLE.

60″ ſeconds	make	1 minute of time
60′ minutes		1 hour
24 hours		1 natural, or real day
7 days		1 week
4 weeks		1 month
13 months or		
365 days		1 year

Note 1. Though the table ſays 13 months, or 365 days make a year, yet it is not truly ſo; for 13 months (allowing 4 weeks to the month) is but 364 days: whereas 13 months, 1 day, 6 hours make a year; for theſe odd 6 hours make 24 hours, or 1 day, every fourth year, which is added to *February*, which has then 29 days, and is called *leap year*: but a true year is 365 days, 5 hours, 48 minutes, and is called a *ſolar year*, being the time that the ſun performs its apparent revolution through the *ecliptic*; but you muſt not buſy yourſelf, and loſe your time about theſe things, which are too hard for you to underſtand at preſent.

Note 2. Though 13 months are ſaid to make a year, and ſervants commonly reckon a month 28 days; yet you are to obſerve, that in trade, and tranſacting buſineſs, by a month is meant a *calendar-month*; that is from any day of the month to the ſame day of the next month: thus, from the 5th of *February* to the 5th of *March*, or from the 18th of *April* to the 18th of *May*, is a month.

Ex-

EXAMPLE 1.				EXAMPLE 2.			
(10)	(13)	(4)	(7)	(10)	(24)	(60)	(60)
Years	*Mon.*	*Wks.*	*Days*	*Days*	*Hrs.*	*Min.*	*Sec.*
27	11	3	6	41	22	50	27
43	9	2	5	17	17	17	15
27	5	1	4	24	15	27	14
36	1	2	3	19	21	19	24
45	10	2	1	27	11	18	19
28	12	1	4	35	14	25	25
209	12	2	2				

Note 3. In *example* 2, you muſt point or dot at every row but the laſt.

And now, *Tyro*, I think you may by this time be perfect in *Addition*, I ſhall only ſet you a few queſtions, and proceed to *Subtraction*.

SECTION. VI.

Containing ſome other uſeful things, neceſſary to be known in number, weight, *and* meaſure.

Note 1. *Of things bought and ſold by the* dozen, ſcore, *or* groſs.

A *Dozen* is 12. A *ſcore* is 20. A *common hundred* is 100. A *long hundred* is 120. A *groſs* is 12 dozen, or 144, and a *great groſs* is 12 times as many, or 1728, *oranges*, and *lemons*, *corks*, *bungs*, and many other things, are bought and ſold by the *dozen*, or *groſs*. *Herrings*, and ſeveral other ſort of fiſh; and all ſorts of *nails*, and many ſuch ſmall things, have ſix ſcore, or 120, to the hundred; but a hundred of *ling cod* is 124 in number; and a hundred books in printing is 104.

Note

Note 2. *Of* Parchment *and* Paper.

1 dozen is 12 ſkins; 5 dozen 1 roll of *parchment*; 24, and ſometimes 25 ſheets make a quire of *paper*, 20 quires 1 ream, and 10 ream 1 bale.

Note 3. *Of the different ſizes of* Books.

Folio is the largeſt of all books, and has but 2 leaves to the ſheet. *Quarto* (marked 4*to*) has 4 leaves to a ſheet. *Octavo*, (or 8*vo.*) is a ſheet doubled into 8 parts; and *duodecimo* (commonly called *twelves*, and marked 12*mo.*) has 12 leaves to the ſheet.

Note 4. *Of* Weight, Measure, &c.

A *faggot* of ſteel 6 ſcore, or 120 *lb.* A *burthen* or *gad* of ſteel 9 ſcore. A *barrel* of anchovies 30 *lb.* A *barrel* of figs from 98 to 300 *lb.* A *barrel* of gun-powder 1 *Cwt.* A *puncheon* of prunes from 10 to 12 *Cwt.* A *ton*, or *fother* of lead 19 *Cwt.* 2 *qrs.* A *quintal* of fiſh 100 in tale. A *ſtone* of iron, ſhot, or horſeman's weight 14 *lb.* A *ſtone* of meal 8 *lb.* A *ſtone* of hemp 32 *lb.* A *ſtone* of glaſs 5 *lb.* A *ſeam* of glaſs 24 *ſtone*, or 120 *lb.* A *keg* of herrings, &c. 60 in number, an *hundred* is 120. A *cade* of ſprats 1000. A *cade* of herrings 500. A *barrel* 1000. A *laſt* 12 barrels, or 12,000. A *laſt* of corn, or rape-ſeed 10 *qrs.* A *laſt* of gun-powder 24 barrels. A *laſt* of leather 20 dickers. A *dicker* 10 ſkins. A *laſt* of hides 12 dozen; of tar 14 barrels; of wool 12 ſacks; of flax, or feathers 1700 *lb.* A *wey* in ſome places is 5 chaldron. A *wey* of meal 6 *qrs.* A *gallon* of train oil $7\frac{1}{2}$ *lb.* A *tun* 252 gallons. A *tun* of ſweet oil 236 gallons. A *load* of hay in ſome places is 25, in others 30 *cwt.* In *London* it is ſold in *truſſes*, containing 56, or 60 *lb.* a truſs, and 36 truſſes to the load. A *load* of *Scotch* coals 1 *cwt.* A *load* of tiles 1000.

Of

Of bricks 500. Bricks in general are 9 inches long, $4\frac{1}{2}$ inches broad and $2\frac{1}{2}$ thick. A *square* of tiling or thatching contains 10 feet every way, that is, 100 feet, and a rod of brick-work 272 feet, 1 quarter; but 272 is reckoned for common work, A *stack* of wood is 3 feet high, 3 feet wide, and 12 feet long; but this is according to the agreement of the master and the workmen.

Tyro. I am extremely obliged to you, sir; but pray am I bound to get these things by heart before I learn *subtraction?*

Philo. Not at all, *Tyro:* learn the rules and the common tables is sufficient. The others I have only added for your further instruction: They are not set you for a task, but for your diversion; and if you now and then read them over, you will soon find the benefit of thus improving your mind; since it will naturally gain you the good will of your parents, your master, and mankind in general, rather than squandering away your time in idleness and mischief, besides the disgrace of living and dying a dunce.

SECTION VII.

Contains some useful and diverting questions to exercise the learner in Addition *only.*

Quest. 1 A Man borrowed of his friend a certain sum of money, and paid him in part 15*l.* 10*s.* and left unpaid 24*l.* 10*s.* What did he borrow? *Ans.* 40*l.*

Rule. *Add the two sums together gives you the answer.*

2. Suppose a person was born in 1709, when will he be fourscore years old. *Ans.* in 1789.

Rule. *Add as many to* 1709 *as will make it* 1789 *is the answer.*

3. 4.

3. *A, B,* and *C* agreed to purchase an estate, *A* laid out, or paid for his part, 140*l.* 10*s.* *B* paid 217 *l.* 10*s.* *C* paid 500*l.* 10*s.* What did the estate cost? *Ans.* 858*l.* 10*s.* Rule. *Add all the sums together.*

4. A factor bought 4 bags of hops, the first, (No. 1.) weighed 2 *C.* 1 *qr.* 14 *lb.* No. 2. 3 *qrs.* 17 *lb.* No. 3, 2 *C.* 3 *qrs.* 13 *lb.* and No. 4. weighed 1 *qr.* 27 *lb.* What is the weight of all? *Ans.* 6 *C.* 2 *qrs* 15 *lb.* *Add all together is the answer.*

5. A shop-keeper having opened a shop, sold the first day as many goods as came to 15*l.* 13*s.* 7*d.* $\frac{1}{2}$, and thus he went on for one week, (*viz.* 6 days). How much did he take in all? *Ans.* 94*l.* 1*s.* 10*d.* $\frac{1}{2}$.

Rule. *Set down* 15. 13. 7 $\frac{1}{2}$, *six times, and add them together gives the answer.*

6. A farmer carried out 100*l.* in order to buy cattle, and brought home but 34*l.* 15*s* 6*d.* What did he lay out? *Ans.* 65*l.* 4*s.* 6*d.* For these two added together make just 100*l.* and so on for any other sum.

DIALOGUE III.

SECTION I.

SUBTRACTION *in whole numbers.*

Tyro. WHAT does *subtraction* teach?

Philo. *Subtraction* teaches to take a *less number* from a *greater* to discover the *difference.* To prove the work, add the *difference* to the *less number*, and if that sum be the same as the *greater*, the work is right, otherwise false.

Tyro. How is *subtraction* performed?

Philo.

Philo. Quite contrary to *addition*; for in *addition* you uſe the word *and*, but *here* you uſe the word *from*. Thus 4 *from* 7, that is, 4 *taken out of* 7, there remains 3, for the *difference*. Again, 5 *from* 10, there remains 5; and 6 *from* 14 there remains 8, and ſo for any other numbers, as you may ſee by the following examples.

	EXAMPLE 1.	Ex. 2.	Ex. 3.	Ex. 4.
	£.	*Yards*	*Ells*	*lbs.*
Greater number	27	95	468	441756
Leſſer nnmber	14	61	162	240256
Difference	13	34	306	201500
Proof	27	95		

The WORK *in words at length.*

In *example* 1. I ſay, 4 *from* 7 there remains 3; and 1 *from* 2 there remains 1. *To prove it,* I add the *difference* to *leſs numbers,* ſaying, 3 *and* 4 is 7, *and* 1 *and* 1 makes 2. In *Ex.* 2. I ſay 1 *from* 5 there remains 4; and 6 *from* 9 there remains 3. *Proof,* I ſay 4 *and* 1 is 5; 3 *and* 6 9: That is, 34 *and* 61 added together make 95, and ſo for the other examples.

Tyro. This is quite plain indeed.

Philo. I ſhall work *example* 3 and 4 in the ſame manner, and leave you to prove them as I have directed. Obſerve then, *Ex.* 3, I ſay, 2 *from* 8 there remains 6; then 6 *from* 6, or 6 taken out of 6, there remains 0, and 1 *from* 4 there remains 3. *Laſtly,* in *Ex.* 4, I ſay, 6 *from* 6, there remains 0; then 5 *from* 5 there remains 0; 2 *from* 7 there remains 5; then 0 from 1, that is nothing taken out of 1, there ſtill remains 1; then 4 *from* 4, there remains 0, and 2 *from* 4, 2.

Note, I ſhall give you an example or two with, and without cyphers, that you may endeavour to avoid that idle cuſtom.

Ex.

Ex. 5. *with cyph.*		*without*	Ex. 6. *with cyph.*	*without*
From	24526	24526	84257	84257
Take	00012	12	84215	84215
Anſ.	24514	24514	00042	42

Tyro. I own that the examples without the cyphers look beſt, and I underſtand very well what you have ſhewn me.

Philo. I ſhall leave you then 3 or 4 examples for practice.

	Yards.	*Ells.*	*Hundreds.*
From	47162	94785	347621
Take	12412	5104	301

Tyro. Now, Sir, be pleaſed to ſhew me how to ſubtract or manage a ſum, when the lower figure is ſometimes larger than the top-one; for I think that it appears difficult to me at preſent.

Philo. Never fear; you will ſoon find it eaſy, if you do but obſerve the following rule.

2. *When the lower figure is larger than the top one, the* Rule *is,*

Take the lower figure out of what you do by, which (in *whole numbers* you know is *ten,* and to that difference, or remainder, add alſo the top figure, and *that* is the true difference, which place under the firſt row. This is what is called *borrowing* in *ſubtraction,* therefore, remember that you are always to carry 1 to the next lower figure for ſo doing: an example or two will make it quite plain.

Ex.

	Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.
	£.	*Ells*	*Yards*	*Bushels.*
From	762	85420	760410	5217624
Take	145	17273	457146	3471276
Difference	617	68147		

Here in *ex.* 1, I ſay 5 out of 2 I cannot have; therefore I take 5 out of 10, (which is what I do by here) and there remains 5, and the top figure 2 makes 7, which I place under the firſt row, and carry 1 to the next lower figure; ſaying, 1 that I carry to 4 makes 5, which taken out of 6, that is, 5 from 6 there remains 1; but now I carry nothing to the next figure, becauſe the lower figure being leſs than the top-one, I had no occaſion to borrow; therefore, I only ſay in the laſt row, 1 from 7, there remains 6: again in *ex.* 2. I ſay, 3 from 0 I can't, but 3 from 10 there remains 7, and 0 is 7 ſtill; then I carry 1 to 7 is 8, from 2 I can't, but 8 from 10 there remains 2, and the top figure 2 makes 4: then I carry 1 to 2, which is 3, ſaying, 3 from 4 there remains 1; but now I do not carry any to the next figure, becauſe I did not borrow, but only ſay, 7 from 5 I can't take, but 7 from 10 there remains 3, and the top 5 makes 8. *Laſtly*, I carry 1 to 1 is 2 from 8 there remains 6.

Tyro. This is plain enough.

Philo. This is the eaſieſt way for a learner, but there is another method which is more practicable and expeditious, if you mind and learn it.

Another way to ſubtract, *when the lower figure is larger than the top-one.*

When you cannot take the lower figure from the top-one, then count the top figure ten more than it really is. Thus, if the top figure be 2 call it 12; if 3 call it 13; if 5 call it 15; if 8 call it 18, *&c.* and then

then take the lower figure from it, and you have the true *answer* or *difference*: but always remember to carry 1 to the next figure for so doing. This is called borrowing *ten*, and your carrying 1 to the next figure is paying of it again.

EPAMPLES.

	£.	*Yards.*	*Ells.*
From	75043	41725	172560
Take	27365	17258	87275
Difference	47678	24467	
Proof	75043	41725	

First, I say 5 from 3 I can't; but calling 3, 13, I say 5 from 13, there remains 8; then I carry 1 to 6 is 7 from 4 I can't, but I say 7 from 14 there remains 7; then I carry 1 to 3 is 4 from 0 I can't, but 4 from 10 there remains 6 Again I carry 1 to 7 is 8 from 5 I can't, but 8 from 15 there remains 7. *Lastly*, I carry 1 to 2 is 3 from 7 and there remains 4.

☞ *Note*, Tho' the bottom figures in *subtraction* may be larger than the top-ones; yet you are to remember the last lower figure is never larger than the top one when there is an equal number of figures.

Tyro. I under stand you very well, sir.

Philo. Then I shall only leave you an example or two, to try at as you have leisure.

More EXAMPLES.

	£.	lb.	*Miles.*
From	417215	62170071	417621700
Take	241729	7210943	631720
Difference			

SEC.

SECTION II.

Of MONEY.

Tyro. HOW is *ſubtraction of money* performed?

Philo. By taking or ſubtracting every denomination in the lower line, out of, or from the upper ſum, as will appear more plain by the 2 following rules.

Rule 1. When the lower figures in the *farthings, pence,* or *ſhillings,* are ſmaller than the top-ones; then only ſubtract or take one from the other, and the remainder is the true *anſwer,* or *difference,* which is placed under each row to which it belongs.

Rule 2. When the lower figures in the *farthings, pence,* or *ſhillings,* are larger than the top-ones; then ſubtract or take the lower figure out of what you do by; that is, take the lower *farthings* out of 4, the lower *pence* out of 12, and the lower *ſhillings* out of 20, taking in the top figure beſides; ſo ſhall this be the true *difference,* or *anſwer*; but remember, that you are always to carry 1 to the next figure for borrowing, as you did in *whole numbers.*

Tyro. Pleaſe to give me an example or two, I ſhall ſoon underſtand it.

Philo. You cannot miſs if you mind the rule well.

	Ex. 1.			Ex. 2.			Ex. 3.		
	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*
From	9	8	11	48	15	5	643	14	$6\frac{3}{4}$
Take	7	5	6	27	11	3	231	11	$2\frac{1}{4}$
Rem.	2	3	5	21	4	2	412	3	$4\frac{1}{2}$
Proof	9	8	11	48	15	5	643	14	$6\frac{3}{4}$

1. I begin at the pence in example 1, ſaying, 6 from 11 pence there remains 5 pence, which I place under the pence: then I ſay 5 ſhillings from 8, there remains 3 ſhillings; and 7 pounds from 9, there remains 2. So the *difference* is 2*l.* 3*s.* 5*d.*

PROOF.

The work is proved like *whole numbers*, by adding the *difference* to the *lower* or *leſs number*: thus, 5 pence and 6 is 11 pence; then 3 ſhillings and 5 make 8 ſhillings; and 2 added to 7 make 9 pounds.

2. In example 2, I ſay, 3 from 5, there remains 2 pence: then 11 from 15, there remains 4; then 7 from 8 there remains 1; and 2 from 4 there remains 2. *Ex.* 3. having farthings, I ſay, 1 farthing from 3, there remains 2 farthings, which I ſet down thus $\frac{1}{2}$; then 2 from 6, there remains 4 pence; then 11 from 14 ſhillings, there remains 3. Laſtly, for the pounds I ſay, 1 from 3, there remains 2: then 3 from 4, there remains 1; and 2 from 6 there remains 4: ſo is the *difference* or *anſwer* 412*l.* 3*s.* 4*d.* $\frac{1}{2}$. Now if you add this 412*l.* 3*s.* 4*d.* $\frac{1}{2}$ to the lower ſum 231*l.* 11*s.* 2*d.* $\frac{1}{4}$ you will find they amount to the ſame as the top ſum, *viz.* 643*l.* 14*s.* 6*d.* $\frac{3}{4}$.

Note. After this manner is every ſum done and proved, in *ſubtraction.*

More examples for practice.

From	471 - 11 - 2$\frac{1}{4}$	576 - 19 - 11$\frac{1}{2}$	409 - 11 - 5$\frac{3}{4}$
Take	120 - 10 - 2	132 - 13 - 4$\frac{1}{4}$	304 - 11 - 3$\frac{1}{2}$
Remains			

2. *When*

2. *When the* shillings, pence, *and* farthings, *are larger in the lower line than in the top-one.*

	EXAMPLE 4.	EXAMPLE 5.	EXAMPLE 6.
	(10) (20)(12)	(10) (20) (12)	(10)(20)(12)(4)
	£. s. d.	£. s. d.	£. s. d.
Lent	10 - 4 - 6	42 - 11 - 6	453 - 13 - 5¼
Received	7 - 10 - 8	17 - 14 - 10	326 - 15 - 8¾
Due	2 - 13 - 10	24 - 16 - 8	126 - 17 - 8½
Proof	10 - 4 - 6	42 - 11 - 6	453 - 13 - 5¼

Here I say 8 pence from 6 pence I can't; but 8 from 12 (which is what I do by at pence) there remains 4, and the top 6 makes 10: then I carry 1 because I borrowed, saying, 1 that I carry to 10 is 11 shillings, which from 4 I can't take; but 11 from 20 (which is what I do by) there remains 9, and the top 4 makes 13. *Lastly*, I carry 1 to 7 is 8 from 10 there remains 2.

PROOF.

I prove these examples the same way as before, by adding the *difference* to the *less sum*, saying, 10 pence, and 8 is 18 pence, that is, 6 pence above a shilling, or 12 pence; and I carry 1 to 13 is 14, and 10 is 24, which is 4 above 20 shillings, or 1 *l.* *Lastly*, I carry 1 to 7 is 8, and 2 makes 10.

Now, *Tyro*, pray try you at *example* 6, where there are farthings.

Tyro. In *example* 6, I say first, 3 farthings from 1 farthing I can't; but 3 farthings from a penny, or 4 farthings, there remains 1, and the top farthing makes 2 farthings, which I set down thus ½: then I carry 1 to 8 pence is 9 pence, which from 5 pence I can't have; but 9 from 12 there remains 3, and the top 5

makes 8 pence: again, I carry 1 to 15 is 16, from 13 I can't; but 16 from 20 there remains 4, and the top 13 is 17 ſhillings. *Laſtly*, I carry 1 to 6 is 7 from 3 I can't; but 7 from 13 there remains 6. Or 7 from 10 there remains 3, and the top 3 is 6: then I carry 1 to 2 is 3 from 5 there remains 2; but now carry none becauſe I did not borrow; therefore I only ſay 3 from 4 there remains 1.

P R O O F.

I prove this, as before directed, ſaying, 2 farthings and 3 is 5 farthings, which is 1 farthing above a penny, and I carry a penny to 8 is 9, and 8 is 17 pence, 5 above 12, and I carry 1 to 17 is 18, and 15 is 33, which is 13 above 20, and I carry 1 to 6 is 7, and 6 is 13, which is 3 above 10, and I carry 1 to 2 is 3, and 2 is 5; then I ſay 1 and 3 is 4.

Philo. You have done very well indeed; I ſhall only ſet you an example or two for practice.

E X A M P L E S *for Practice.*

	£.	s.	d.		£.	s.	d.
Borrowed	217	10	0	*Debtor*	1000	00	0
Paid	109	15	$8\frac{1}{4}$	*Creditor*	910	15	$6\frac{1}{2}$
To pay				*Balance*			

Tyro. Theſe I can do very well; therefore wiſh you would ſhew me how to balance an accompt, or make out a ſmall reckoning.

Philo. That I will.

3. *Practical queſtions for buſineſs.*

Queſt. 1. *A* borrowed of *B* ——— 100 - 00 - 0

	£.	s.	d.
A paid him at one time	21	14	6
At another 10 guineas, *viz.*	10	10	0
At another 20 guineas, *viz.*	21	00	0
At another time	5	15	0
Sold him goods amounting to	18	14	9
	77	14	3

In all 77 - 14 - 3

What is ſtill due to *B*? *Anſwer* 22 - 5 - 9

Rule. I add up together all the ſums that *A* paid at different times, and find they amount to 77*l.* 14*s.* 3*d.* which I place under 100*l.* and ſubſtracting it therefrom, I find that there is ſtill due to *B* 22*l.* 5*s.* 9*d. anſwer.*

Queſt. 2. *A* Blackſmith *delivered a bill to a* Farmer *of* 45*l.* 14*s.* 6*d.* ¼, *and the* Farmer *has paid him in part as under.*

	£.	s.	d.
By caſh ———	20	00	0
By 10 buſhels of malt ———	1	15	0
By a load of hay ———	1	16	0
By meat at ſeveral times ———	2	7	6
By 14 buſhels of oats ———	2	11	0
By 20 buſhels of wheat ———	4	7	3
	32	16	9

	£.	s.	d.
Blackſmith's bill ———	45	14	6¼
Farmer's bill ———	32	16	9
Balance due to the *Blackſmith*	12	17	9¼ *anſwer.*

Queſt. 3. A *ſteward* collected as much money for rents as amounted to ———— £. 4000
And remitted to his maſter as under.

By caſh at one time ———	500
At another ——— ———	500
At another ——— ———	210
At another ——— ———	420
By tax bills diſcounted ———	41 - 15 - 6¼
By repairs done to the eſtate ———	641 - 14 - 9
By other charges ———	15 - 13 - 6
Laid out in all £.	2329 - 3 - 9¼
Which taken from the top £. 4000	
there remains due to his maſter	1670 - 16 - 2¾

Tyro. I underſtand you well.

Philo. Then I will leave you one *queſtion* for practice, and you may do the work at leiſure.

Queſt. 4. Two perſons *A* and *B* have a reckoning to ſettle as follows: *A* lent *B* £. 300, and ſome time after lent him 100 guineas more: *B* paid him at 3 ſeveral times, each 100 guineas, and at another time gave him a *draught* or *note* upon *C*, for £. 50, and ſold him as many goods as came to £. 34 : 4 : 6 : now I demand how the balance ſtands between them. *Anſ.* there is ſtill due to *A* £. 5 : 15 : 6. To prove it,

Rule. Add the two ſums that *A* lent to *B* together, and you will find them £. 405; then ſet all that *B* paid in caſh, and the *draught*, and goods, all under one another, and add them together, which makes £. 399 : 4 : 6: this being done, ſet £. 399 : 4 : 6 under £. 405, and *ſubtract* it therefrom, and you will find there remains £. 5 : 15 : 6 due to *A* on balance.

Tyro. I will try it directly, and am ſure it is eaſy enough, but your anſwer will be ſome help to me I own.

Philo. As every accompt of *debtor* and *creditor* is ſettled after this manner, I ſhall give no more examples, expecting

expecting by this time you are capable of setting yourself questions; and if you are not, it is highly necessary to look over the examples once more, then will you be duly qualified for those questions I shall set you at the end of *subtraction:* and therefore we will proceed to *weights* and *measures*, where you will find some examples done for your instruction, and some left undone for your practice.

AVOIRDUPOIS WEIGHT.

	Tons	C.	qr.	lb.		C.	qr.	lb.	oz.	dr.
Bought	17	11	1	15	*Bought*	47	1	17	11	14
Sold	14	17	3	21	*Sold*	19	0	25	10	15
Unsold	2	13	1	22	*Unsold*					

Now, *Tyro*, I shall shew you this first *example*, but no more; because all other sums in every one of the rules are done in the same manner, and there is no occasion for further instruction.

First, I say 21 from 15 I can't, but 21 from 28 (which is what I do by at *lbs.*) there remains 7, and 15 makes 22: then I carry 1 to 3 is 4 from 1 I can't; but 4 from 4 (which is what I do by at *qrs.*) there remains 0, but the top 1 is 1, which I set down. Again, I carry 1 to 17 is 18 from 11 I can't; but 18 from 20 there remains 2, and the top 11 makes 13. *Lastly*, I carry 1 to 4 is 5 from 7, there remains 2, and the work is done.

Tyro. I thank you Sir; examples now whithout words will be sufficient.

Note, If you forget what you do by, turn to your tables in *addition*.

TROY-WEIGHT.

	lb.	oz.	dw.	gr.		lb.	oz.	dw.	gr.
Bought	14	5	15	14	*Bought*	214	10	17	11
Sold	9	9	13	17	*Sold*	109	10	17	15
Rem.	4	8	1	21	*Remains*				

DRY-MEASURE.

	Loads	bush.	pecks	pints		Chal.	bush.	pecks.
Bought	42	17	2	12	*Bought*	291	21	2
Sold	17	34	2	14	*Sold*	173	27	3
Unsold	24	22	3	14	*Unsold*			

WINE-MEASURE.

	Butts	hhds.	gall.	pints		Hhds.	gall.	pints.
Bought	14	1	47	0	*Bought*	64	35	1
Sold	9	0	51	5	*Sold*	17	46	2
Unsold	5	0	58	3	*Unsold*			

WINCHESTER-MEASURE.

	Bar.	gall.	pints		Butts	hhds.	bar.	gall.
Brewed	124	21	4	*Brewed*	21	1	1	20
Sold out	92	27	6	*Sold out*	15	1	0	26
Unsold	31	29	6	*Unsold*				

Note, I have reckoned 36 gallons to the barrel, that being customary in selling beer in most places, as I said before.

CLOTH-

CLOTH-MEASURE.

	Yds. qrs. nails	*Ells Eng. qrs. nails*	*Ells Fl. qrs. N.*
Bought	147 - 1 - 0	47 - 1 - 3	47 - 2 - 0
Sold	96 - 3 - 2	18 - 2 - 3	29 - 2 - 1
Rem.	50 - 1 - 2		

LONG-MEASURE.

	Deg.	*leag.*	*miles*	*fur.*	*rods*	*yds.*	*feet*	*inc.*	*B.corns*
From	471	14	1	3	17	2	1	10	1
Take	167	17	1	5	21	3	1	10	2
Rem.	303	16	2	5	35	$3\frac{1}{2}$	2	11	2

LAND-MEASURE.

	Acres roods poles yds.	*Acres roods poles*
From	471 - 2 - 15 - 3	47 - 1 - 15
Take	196 - 2 - 26 - 4	19 - 2 - 15
Rem.	274 - 3 - 28 - $4\frac{1}{2}$	

TIME.

	Yrs. mths. wks. days hours.	*days hours min. sec.*
From	219 - 10 - 3 - 3 - 17	45 - 17 - 25 - 17
Take	193 - 11 - 3 - 4 - 20	19 - 21 - 43 - 35
Rem.	25 - 11 - 3 - 5 - 21	

Note, In the first example I reckon 13 months to a year.

Tyro. Sir, I am obliged to you for the pains you have taken; I understand you very well, and will try at all those sums you have left undone.

Philo. That is enough, we will finish *subtraction* then with *Section* 3.

SECTION. III.

Containing some useful QUESTIONS *to exercise the learner in both* RULES.

Quest. 1. KING *Harry* the eighth died 1547; I demand how many years it is, this being now 1768 *Answer* 221 years.

Quest. 2. Suppose this present year 1768, you were 19 years old, what year was you born in. *Ans.* 1749.

Quest. 3. A boy had 1000 marbles, and he lost at 3 different times at play, each 175, and at another time 150; how many has he still in hand. *Ans.* 325.

Rule. Set down 175 three times, and 150 under that; and then add all of them together, and take the sum out of 1000, and the answer will be 325 left. This *rule* serves for all that follows.

MONEY.

4. *A* lent *B* £. 500, and *B* paid him 4 times, each £. 120, 10*s*. What is due to *A*? *Ans.* £. 18.

5. What sum of money must I add to £. 58-14-6$\frac{3}{4}$, to make it up £. 100? *Ans.* £. 41-5-5$\frac{1}{4}$.

6. *A proof to question* 5. What sum must I take out of £. 100, to have the remainder £. 58 - 14 - 6$\frac{3}{4}$. *Answer* £. 41 - 5 - 5$\frac{1}{4}$.

7. A collector of the excise received £. 2040-14-5$\frac{3}{4}$, and remitted, or paid at 3 several times, each £. 500, and at another time 100 guineas: what has he still in Hand? *Ans.* £. 435-14-5$\frac{3}{4}$.

AVOIR-

AVOIRDUPOIS-WEIGHT.

8. Bought 6 *ton*, 14 *Cwt*. of iron, and ſold at one time, 3 *ton*, 11 *C*. 2 *qrs*. and at ſeveral other times, ſold by retail 15 *C*. 3 *qrs*. 17 *lb*. What remains unſold? *Anſ*. 2 *ton*, 6 *C*. 2 *qr*. 11 *lb*.

TROY-WEIGHT.

9. A gentleman delivered to a ſilverſmith 2 *lb*. 5 *oz*. 11 *dwts*. of ſilver; and he received a ſilver cup, which weighed 11 *oz*. 14 *dwts* and at another time, 6 large ſpoons, weighing 1 *lb*. 2 *oz*. 3 *dwts*. 14 *grs*. What weight of ſilver has the ſilverſmith ſtill in hand? *Anſ*. 3 *oz*. 13 *dwts*. 10 *grs*.

LIQUID-MEASURE.

10. A common brewer has 2 goils of new beer, containing 32 butts, 2 barrels, 24 gallons of ſtrong; and 15 butts 1 barrel of ſmall. And he ſent to one victualler 8 butts 2 barrels, to another 11 butts, and ſtarted into his own ſtore-houſes 5 butts, 2 barrels, 27 gallons of the ſtrong; and of the ſmall 7 butts, 2 barrels; he alſo ſold out by retail of the ſmall, 4 butts, 23 gallons; I demand what remains of the brewing, both of the goil of ſtrong, and the goil of ſmall? *Anſ*. 7 butts, 33 gallons of ſtrong, and 3 butts, 1 barrel, 13 gallons of ſmall.

☞ *Note*, I have added the anſwers, that you may know when you are right or wrong: if you cannot do all theſe queſtions as yet, it is of no great ſignification; you muſt not ſtop your progreſs in going forward, becauſe of that. Now follows

DIA-

DIALOGUE IV.

SECTION I.

MULTIPLICATION.

Tyro. WHAT is *multiplication?*

Philo. Multiplication is a compendious or short way of *addition*, and teaches us to tell the product of any sum in one line only, which would require several *additions*.

Tyro. What else is required?

Philo. There are three things to be carefully observed in *multiplication*.

First. The *multiplicand*, which stands a-top, and is that number given to be multiplied.

Secondly. The *multiplier*, or sum you multiply by.

Thirdly. The *product* or *answer*, which is the *multiplicand* multiplied by the *multiplier*.

Tyro. Please to explain this a little more.

Philo. Let us take any two numbers, suppose 8 and 3, I set down 8, and place 3 under it; so is 8 the *multiplicand*, 3 is the *multiplier*, and 24 is the *product*, or *answer*, because 3 times 8 make 24. Is this plain enough.

Tyro. Yes, sir, I understand you now.

Philo. Very well, *Tyro*, then I would have you get the following table by heart, by all means, notwithstanding it is too customarily omitted.

MULTIPLICATION TABLE.

Once 1 is 1

2 times, or twice	2	is	4
	3		6
	4		8
	5		10
	6		12
	7		14
	8		16
	9		18
3 times	3	is	9
	4		12
	5		15
	6		18
	7		21
	8		24
	9		27
4 times	4	is	16
	5		20
	6		24
	7		28
	8		32
	9		36
5 times	5	is	25
	6		30
	7		35
	8		40
	9		45
6 times	6	is	36
	7		42
	8		48
	9		54
7 times	7	is	49
	8		56
	9		63
8 times	8	is	64
	9		72
9 times	9	is	81
10 times	10		100
11 times	11		121
12 times	12		144

Table of twelves.

12 times	2	is	24
	3		36
	4		48
	5		60
	6		72
	7		84
	8		96
	9		108
	10		120
	11		132
	12		144

Note 1. You ſhould be very careful to get the *table* by heart, as I ſaid before: ſaying thus, 2 times 2, or twice 2 is 4, twice 3 is 6, twice 4 is 8, twice 5 is 10, *&c.*

Note 2. You ſhould alſo at leiſure learn to *box* the *table* well: that is, to ſay it backwards, forwards, or any way. Thus you ſee twice 5 is 10, that is, 5 times 2 is 10: Again, 3 times 9 is 27; ſo alſo is 9 times 3 the

the same. So 6 times 12 is the same as 12 times 6, and 5 times 9 the same as 9 times 5.

1. *To multiply by single figures.*

Rule. Multiply every figure in the *multiplicand* by the figure in the *multiplier*, carrying 1 for every 10, as in *addition of whole numbers*, and you have the *product* or *answer*.

EXAMPLE 1.		EXAMPLE 2.	
Multiplicand	17	*Multiplicand*	78
Multiplier	5	*Multiplier*	6
Product	85	*Product*	468

I begin with the *multiplier* 5, saying, 5 times 7 is 35, and set down the 5, and I carry 3; then I say 5 times 1 is 5, and 3 that I carried is 8; so is the *product*, or *answer*, 85. Again,

In *Ex.* 2. I say, 6 times 8 is 48; that is 8, and I carry 4; then 6 times 7 is 42, and 4 I carried is 46: so is this *product* 468.

To prove the work by Addition.

I set the *multiplicand* 17 down 5 times; and the *multiplicand* 78 I set down 6 times, and adding them together, I find 85, and 468, as before; which you may try on your slate at leisure.

Tyro. I see plainly that *multiplication* saves the trouble of many *additions*.

Philo. There is another way to prove *addition*; but as it is of no manner of signification, we will lay aside those puzzling curiosities, and proceed to

Ex-

EXAMPLE 3.	EXAMPLE 4.	EXAMPLE 5.
Multiply 472 *by* 8	24970 *by* 9	and 590407 *by* 7
Answer 3776	*Ans.* 224730	*Ans.* 4132849

Tyro. I say 8 times 2, (or twice 8) is 16, 6 and I carry 1; then 8 times 7 is 56, and 1 is 57; that is, 7, and I carry 5; then 8 times 4 is 32, and 5 I carried is 37. In *Ex.* 4, I say, 9 times (0) nothing is 0; then 9 times 7 is 63, that is 3, and I carry 6; then 9 times 9 is 81, and 6 I carried is 87, that is 7, and I carry 8; then 9 times 4 is 36, and 8 is 44, that is, 4 and I carry 4; and *lastly*, 9 times 2 is 18, and 4 I carried is 22. Also, in *Ex.* 5, I say, 7 times 7 is 49, that is 9, and I carry 4; 7 times 0 is 0, but 4 is 4 still; then 7 times 4 is 28, that is 8, and I carry 2; then 7 times 0 is 0, but 2 is 2; then I carry none, but say, 7 times 9 is 63, that is 3, and I carry 6, and *lastly*, 7 times 5 is 35, and 6 is 41.

Philo. Very well done indeed! I leave these two examples for practice.

Multiply 49997296 *by* 9	and 94171470 *by* 8
Answer	*Answer*

Tyro. Please to give me an example or two to *multiply* by 12 in one line; for I know it is done much quicker, and it is as easy as to make two of it.

EXAMPLES *with Twelves.*

Multiply 42576 *by* 12	and 994079 *by* 12	and 99807 *by* 12
Answer 510912	*Ans.* 11928948	*Ans.*

First,

First, 12 times 6 is 72, that is 2, and I carry 7; then 12 times 7 is 84, and 7 I carried is 91, that is 1, and I carry 9; then 12 times 5 is 60, and 9 I carried is 69, that is 9, and I carry 6; then 12 times 2 is 24, and 6 is 30, that is 0, and I carry 3; and *lastly*, 12 times 4 is 48, and 3 is 51. And thus by having the *table of twelves* perfectly by heart, every sum will be easy to you.

Tyro. It is easy enough, I see: please now to shew me how you multiply by 2, 3, 4, or more figures.

Philo. This you will soon understand.

2. *Of multiplying by several figures.*

Rule. When there are several figures in the *multiplier*, then you begin at the first figure in the units place, and multiply it through, or into every figure of the *multiplicand*, as you have done before. This being done, multiply every figure in the *multiplicand* by the second figure of the *multiplier*; only observe to set the first figure of the second row under the *tens place* of the first row; and thus you go on with the third figure in the *multiplier*, placing the *units place* of each succeeding row under the *tens place* or the row that is above it, till you have gone through every figure of the *multiplier*. Then draw a line under all the rows of figures, and add them together, and the sum is the true *product*, or *answer*.

Note, Be sure you remember that you set your figures right under one another, or else in a large sum you will be puzzled to add the work together.

Ex-

EXAMPLES *with several figures.*

EXAMPLE I.		PROOF.	
Multiply	89	*Multiply*	47
by	47	*by*	89
	623		423
	356		376
Answer	4183	*The same*	4183

In *example* 1, I say, 7 times 9 is 63, that is 3, and I carry 6; then 7 times 8 is 56, and 6 I carried is 62, which I set down; so is the first line or row finished: Then I take the second figure of the *multiplier*, saying, 4 times 9 is 36, which 6 I set under the 2, or *tens place* of the first line and carry 3; and then I say, 4 times 8 is 32, and 3 that I carried is 35, placing the 5 under the 6, and the 3 quite out towards the left-hand. *Lastly*, I add these up in order, as they stand, saying, 3 is 3; then 6 and 2 is 8; again, 5 and 6 is 11, that is 1, and I carry 1 to 3 is 4.

To prove MULTIPLICATION.

It is a common way to prove *multiplication* by the cross; but it is subject to so many errors, that in short, it is no proof at all to a learner, but rather a corruption. The best way therefore is this: take the *multiplicand* and set it below, and the *multiplier* a-top, that is, change the *multiplicand* into the *multiplier*, and proceed as before directed, and if the *product* be the same as before, your work is entirely right. See the next examples wrought at large both ways.

Ex-

EXAMPLE 3.

Multiplicand	895
Multiplier	798
	7160
	8055
	6265
Product	714210

PROOF.

Multiplicand	798
Multiplier	895
	3990
	7182
	6384
The same	714210

First, I say, 8 times 5 is just 40; therefore I set down the 0, and I carry 4; then 8 times 9 is 72, and 4 I carried is 76, that is 6, and I carry 7; then 8 times 8 is 64, and 7 I carried is 71. Now I take the second figure, saying, 9 times 5 is 45; which 5 I place under the second figure, or *tens place* of the first line, and carry 4, saying, 9 times 9 is 81, and 4 is 85, which 5 I place under the figure 1, and carry 8 to the next figure, saying, 9 times 8 is 72, and 8 that I carried is 80. And now I come to the last figure of the *multiplier*, saying, 7 times 5 is 35, which 5 I place under the second figure of the last line, and carry 3 to the next figure; saying, 7 times 9 is 63, and 3 I carried is 66, that is, 6 and I carry 6; then 7 times 8 is 56, and 6 is 62. *Lastly*, I add these up in order as they stand, and find the *product* 714210.

The proof of this example is worthy your observation, *Tyro*; for be the sum ever so large, if you change the *multiplicand* into the *multiplier's* place, and multiply right, you will find the *product* always the same.

Tyro. I see it plainly, sir; and I could not have thought *multiplication* had been so easy.

Philo. Nothing easier, when the *table* is once well learned. I shall now give you two more examples, and leave the rest undone for practice.

Ex-

EXAMPLE 4.		EXAMPLE 5.	
Multiply	913876	and	749076
by	8759	*by*	1795
	8224884		3745380
	4569380		6741684
	6397132		5243532
	7311008		749076
	8004639884		1344591420

EXAMPLES *for practice.*

Multiply	4567879	*Multiply*	9567950
by	45769	*by*	32796

☞ *Note*, Remember, *Tyro*, that when you multiply by ſeveral figures, that you always ſet the firſt figure of the ſecond line under the ſecond figure of the firſt line, and the firſt figure of the third line under the ſecond figure of the ſecond line: and thus go on, leaving one figure out towards the right-hand every line, ſetting the next line one figure more towards the left-hand, and every figure under one another in their proper place.

Tyro. I underſtand you well, ſir.

Philo. Then I will ſet you but one example more, which you may prove yourſelf.

Multiply

Multiply	987654321
by	123456789
	8888888889
	7901234568
	6913580247
	5925925926
	4938271605
	3950617284
	2962962963
	1975308642
	987654321
Answer	121932631112635269

PROOF.

Multiply	123456789
by	987654321

3. *Of cyphers between the figures, &c.*

The most difficult thing in this rule, is, when the *multiplier* has some figures, some cyphers; but if you be careful to mind the work of an example or two, you will soon understand it, as I shall explain them in words.

Example 1.

Multiply	49657
by	30705
	248285
	3475990
	1489710
Answer	1524718185

Example 2.

Multiply	7564965
by	5003007000
	52954755000
	2269489500
	3782482500
Answer	37847572849755000

Note,

Note, Pray obſerve, that you read the following inſtructions once or twice over, if you do not underſtand the work.

Firſt, I begin at the 5, and multiply all the top figures by it, as before: now, as the next figure of the *multiplier* is a cypher, it is of no ſignification to multiply by *that*, becauſe it will produce nothing but a line of cyphers: therefore, I bring it down, that is, I ſet a cypher under the figure 8, and then begin to multiply by the 7, ſaying, 7 times 7 is 49, 9 and I carry 4; which 9 I ſet down in the ſame line, on the left-hand of the cypher, and go on ſtill to multiply by the 7; then as the next figure is a cypher again, I ſet down a cypher under it, as I did before, leaving the 9, and the cypher which ſtands by it, in the laſt line, both ſtanding out towards the right-hand. This being done, I only multiply by the 3, and ſet it in the ſame line, and add all up, and the work is done. Again.

In *example* 2, there are 3 cyphers at the beginning of the *multiplier*, therefore I ſet 3 cyphers under the line, right under them: then multiply by the 7 as in all other ſums; ſaying, 7 times 5 is 35, *&c.* Then I come to 2 cyphers more in the *multiplier*, and therefore, ſet 2 cyphers right under them, and then multiply by the 3, and place the product by the ſide of the 2 cyphers: then, as there are two cyphers, I again ſet down two cyphers, thus 00, under them, and then multiply by the 5, placing the product by the ſide of them. *Laſtly*, I add the ſum up as it ſtands, and it is done.

Note, Theſe examples are proved like the others, by ſetting the *multiplier* a top, and the *multiplicand* under it.

More

More examples for practice.

Multiply	570965	and	71900072
by	2400500	*by*	5000960000

And now *Tyro*, I ſhall finiſh this *ſection* with ſhewing you ſomething of *contractions*, or *compendiums*.

4. *Of* CONTRACTIONS.

When any number of figures are given to be multiplied by 10, 20, 30, 40, 80, 90, 120, 1200; then ſet the cyphers out, and multiply by the figure, or figures, and place the *product* by the ſide of the cyphers, as before, gives the *anſwer*.

Multiply	275	and	3275	and	6759
by	20	*by*	90	*by*	1200
Anſwer	5500	*Anſ.*	294750	*Anſ.*	8110800

2. When you are to multiply by 1, the *anſwer* will be the ſame as the *multiplicand* itſelf is. If you are to multiply by 10, add 1 cypher to the *multiplicand*; if by 100, add 2 cyphers; if by 1000, add 3 cyphers.: See the work two ways.

Multiply	1753		1753		1753
by	100		1000		10000
Anſwer	175300	*Anſ.*	1753000	*Anſ.*	17530000

Or

Or rather thus:

Multiply 1753 *by* 1	*answer* the ſame, *viz.*		1753
by 10	add 1 cypher,	*anſ.*	17530
by 100	add 2 cyphers,	*anſ.*	175300
by 1000	add 3 cyphers,	*anſ.*	1753000
by 10000	add 4 cyphers,	*anſ.*	17530000

And thus for as many cyphers as you pleaſe.

Tyro. The examples are quite plain and eaſy.

Philo. I am glad that you underſtand them; for now I ſhall ſhew you *that* which will ſerve you for all common buſineſs, in caſting up any ſort of goods; and pray be careful to mind it well; for there is not a more uſeful thing in all common *arithmetic*, it being a ſhort way of working the *rule of* 3, without *diviſion*.

Queſtions *performed by* Multiplication.

Tyro. How are theſe queſtions performed?

Philo. By the following rule, *viz.*

Multiply the price by the given number, or quantity, and carry 1 for every 4, 12, and 20, as you do in *addition of money*, and 1 for every 10 in the pounds, as in *whole numbers*.

An example or two well explained will ſoon make it eaſy to you.

	£.	s.	d.	
Queſt. 1. What coſt 3 ells, at	0 -	5 -	3	an ell?
			3	
	£. 0 -	15 -	9	*Anſ.*

Here I multiply the price by the quantity; ſaying, 3 times 3 is 9 pence, which I place under the pence; and then I ſay 3 times 5 ſhillings is 15, which I place under the ſhillings, and the anſwer is 15 ſhillings, and 9 pence.

☞ You

☞ You muſt remember, that 3 ells, at 5*s.* 3*d.* an ell, is the ſame as 3 yards, or 3 groſs, or 3 gallons, or any other name whatſoever; for it is only 3 times 5 ſhillings, and 3 pence,

2. What comes 5 groſs of any thing to, at £. 0 - 6 - 8 a groſs?

£. 0 - 6 - 8	a groſs?
5	
£. 1 - 13 - 4	*anſ.*

Here I multiply the price by 5, ſaying, 5 times 8 is 40 pence, (which is 3 ſhillings and 4 pence) therefore I ſet down 4 under the pence, and carry 3, and then 5 times 6 ſhillings is 30, and 3 I carried is 33 ſhillings, which is 1*l.* 13*s.* that is 13, and I carry 1; and then I ſay, 5 times 0 is 0, but 1 is 1.

3. How much does 7 times 8 ſhillings and 9 pence amount to? Or,
What coſt 7 reams of paper at £. 0 - 8 - 9 a ream?

£. 0 - 8 - 9	a ream?
7	
£. 3 - 1 - 3	*Anſ.*

Here I ſay, 7 times 9 is 63 pence, which is 5 ſhillings, and 3 pence, that is 3, and I carry 5; then 7 times 8 is 56 ſhillings, and 5 I carried is 61; that is 3*l.* 1*s.* or, which is the ſame, it is 3 twenties, and 1 over; therefore, I ſet 1 under the ſhillings, and carry 3 to the pounds, ſaying, 7 times 0 is 0, but 3 is 3: ſo is the anſwer 3*l.* 1*s.* 3*d.*

Note, Tho' I have ſet cyphers in the place of pounds in theſe examples, and have alſo ſet £. *s.* *d.* over the ſums, yet it is better to leave out the cyphers, and more practicable, to make only a great £. before the pounds, as in the following examples.

4. What

4. What cost 9 gallons of brandy, or any thing else, at ——————— £. — 8 $3\frac{1}{4}$ a gallon

$$\begin{array}{r} 8\ 3\frac{1}{4} \\ 9 \\ \hline £.\ 3 - 14\ 5\frac{1}{4} \end{array}$$

Here I multiply the price by 9, beginning at the farthing, say, 9 farthings is 2 pence farthing, that is 1 farthing, and I carry 2 pence; then 9 times 3 is 27 pence, and 2 pence I carried is 29 pence, which is 2 shillings, and 5 pence; therefore, I set down 5 under the pence, and carry 2, saying, 9 times 8 is 72 shillings, and 2 I carried is 74. Now 74 shillings is 3 *l.* 14 *s.* therefore, as there is nothing in the pounds to multiply, I set down 3 *l.* 14 *s.* *viz.* the 14 under the shillings place, and the 3 towards the left-hand.

Tyro. This is very pretty, and very easy: please to try me with an Example.

Philo. There is no doubt but you will do it.

5. A Gentleman gave 10 poor Widows three half crowns a-piece: how much did he give in all? or thus, What cost 10 bushels of any thing, at a bushel } £. — 7 — 6

$$\begin{array}{r} 7 - 6 \\ 10 \\ \hline £.\ 3 - 15 - 0 \end{array}$$

Tyro. I set down 3 half crowns, that is, 7 *s.* 6 *d.* and multiply it by 10, saying, 10 times 6 is 60 pence, that is just 5 shillings; therefore, I set down nothing (0) and carry 5; then I say, 10 times 7 is 70, and 5 I carried is 75 shillings, which is 3 *l.* 15 *s.* So is the answer £. 3 — 15.

6. What comes 12 months wages to, at } £. 1 - 8 - 6¼ a month?

12

Anſ. £. 17 - 2 - 3

Or how much is 12 times £. 1 - 8 - 6¼

Here I ſay, 12 farthings is 3 pence, that is 0, and I carry 3; then 12 times 6 is 72, and 3 I carried is 75 pence, which is 6*s.* and 3 pence; I therefore ſet down 3, and carry 6; then I ſay, 12 times 8 is 96 ſhillings, and 6 I carried is 102 ſhillings, which is 5*l.* 2*s.* ſet down 2, and carry 5; then I ſay 12 times 1 is 12, and 5 I carried is 17*l.* So is the anſwer £. 17 - 2 - 3.

2. *Of double figures in the ſhillings.*

When you come to double figures, ſuch as 14, 17, 18 or the like, multiply them like *whole numbers*, and then caſt out the twenties, that is, count how many times 20 you can have in the number, and it is done. There are ſeveral other ways I ſhall ſhew you, and always take *that* which appears eaſieſt to you, till perfect.

7. What coſt 5 ſheep, at £. 1 - 17 - 6 each?
Or, what is 5 times £. 1 - 17 - 6

5

Anſ. £. 9 - 7 - 6

Here I ſay 5 times 6 is 30 pence, is 2*s.* 6*d.* that is 6 and I carry 2; then I multiply the whole 17 by 5, ſaying 5 times 7 is 35, and 2 I carried is 37, 7 and I go 3, then 5 times 1 is 5, and 3 is 8, that is 87 ſhillings, which is 4*l.* 7*s.* that is 7, and I carry 4; then 5 times 1 is 5, and 4 is 9.

Or thus:

I multiply firſt the 7 by the 5, ſetting down the product on a ſlate, or a piece of paper, ſaying 5 times

7 is 35, and 2 I carried is 37 shillings, or 1 *l.* 17 *s.* then I say 5 times 10 is 50 shillings, or 2 *l.* 10 *s.* and add this to 1 *l.* 17 *s.* it makes 4 *l.* 7 *s.* as before. See the next example.

8. What cost 7 gallons, at £. 1 - 15 - 9¼ a gallon?

	7
	£. 12 - 10 - 4¾ *Ans.*

Here I say, 7 farthings is 1 penny, 3 farthings, therefore, I set down ¾ and carry 1; then 7 times 9 is 63, and 1 is 64 pence, that is 5 *s.* and 4 *d.* 4 and I carry 5; then 7 times 5 is 35, and 5 is 40, 0 and I carry 4; 7 times 1 is 7, and 4 is 11, that is, 110 shillings in all, which is 5 *l.* 10 *s.* that is 10, and I carry 5; and *lastly*, 7 times 1 is 7, and 5 is 12.

Or thus, 7 times 5 is 35, and 5 is 40 shillings that is 2 *l.* which I set down any where on a slate; then I say, 7 times 10 shillings is 3 *l.* 10 *s.* Now 3 *l.* 10 *s.* and 2 *l.* is 5 *l.* 10 *s.* as before. Again,

9. What cost 12 quarters of malt, at a quarter } £. 1 - 14 - 6

	12
	Ans. £. 20 - 14 -

Here I say 12 6 pences is 6 shillings, that is 0, and I carry 6; then 12 times 4 is 48, and 6 is 54, 4, and I carry 5; 12 times 1 is 12, and 5 is 17; that is 174 shillings, which is 8 *l.* 14 *s.* that is 14, and I carry 8; then 12 times 1 is 12, and 8 is 20 pounds. Or,

By the second way, 12 6 pences is 6 shillings, as before, 0, and I carry 6; then 12 times 4 is 48, and 6 is 54, that is 2 *l.* 14 *s.* then 12 times 10 shillings is 6 *l.* Now 6 *l.* and 2 *l.* 14 *s.* is 8 *l* 14 *s.* as before, that is 14, and I carry 8; then 12 times 1 is 12, and 8 is 20.

Tyro. I underſtand both the ways very well.

Philo. Take which you think is eaſieſt, they both anſwer the ſame end.

Tyro. But ſuppoſe I have a larger quantity than 12, how muſt I multiply the price by that number?

Philo. By the following rule.

3. *When there are 2 figures to multiply by.*

Rule. When the given quantity is more than 12, and contains any ſuch numbers as can be found in the *Multiplication-table*, then find any 2 numbers, which, when multiplied together, will make the given number; and then multiply the given price by any one of thoſe figures, and that product multiply by the other number, and that product is the anſwer. Thus, ſuppoſe I am to multiply by 15, I find 3 times 5 is 15; therefore, I multiply by the 3 firſt, and ſet down the product; then I multiply this product by 5, and that gives me the anſwer for 15; becauſe 3 times 5, or 5 times 3 make that number. Again, ſuppoſe I were to multiply by 24, I find 4 times 6 is 24; therefore, I firſt multiply by 4, and then multiply that product by 6, which is the anſwer: or otherwiſe, as 3 times 8 is 24, I may therefore multiply firſt by 3 and then by 8. So if the number be 63, I find 7 times 9, or 9 times 7 is 63; therefore, I multiply the price by any one of theſe figures, and that product I multiply by the other for the true anſwer.

10. What coſt 15 ells of holland, at £. - 7 - 6 an ell?

Here 3 times 5 is 15, multiply firſt by			3	
The price of 3 ells —— —	£. 1	- 2	- 6	
Multiply by 5			5	
Price of 15 ells —— ——	£. 5	- 12	- 6	*Anſ.*

Work

Work at Length.

First, I say 3 six-pences is 1 *s.* 6 *d.* that is 6 and I carry 1; then 3 times 7 is 21, and 1 I carried is 22 shillings, which is 1 *l.* 2 *s.* So is the price of 3 ells 1 *l.* 2 *s.* 6 *d.* Now I multiply this 1 *l.* 2 *s.* 6 *s.* by 5 (because 5 times 3 is 15) saying, 5 times 6 is 30 pence, is 2 *s.* 6 *d.* or 5 six-pences is 2 *s.* 6 *d.* that is 6, and I carry 2; then 5 times 2 is 10, and 2 is 12, which I set down. *Lastly*, I say, 5 times 1 is 5; so the answer for 15 ells, at 7 *s.* 6 *d.* an ell, is £. 5 - 12 - 6.

11. What cost 24 chaldrons of coals, at } £. 1 - 11 - 6 a chaldron?

4 times 6 is 24, first multiply by	4
The price of 4 chald.	6 - 6 - 0
6	6
The price of 24 Chald.	£. 37 - 16 - 0 *Ans.*
Proved another way —	£. 1 - 11 - 6
3 times 8 is 24, multiply by	3
Price of 3 chal.	4 - 14 - 6
Multiply by 8	8
Price of 24 chald.	£. 37 - 16 - 0 as above

12. What cost 35 sheep, at	£. 1 - 15 - 7 each?
5 times 7 is 35, multiply first by	5
Price of 5	8 - 17 - 11
7	7
Price of 35	£. 62 - 5 - 5 *Ans.*

 13. What

13. What cost 48 load, at £. 3 - 10 - 6 a load?
6 times 8 is 48, multiply by 6

Price of	6		21 - 3 - 0
Multiply by	8		8
Price of	48	£.	169 - 4 - 0

14. What cost 72 pieces of Holland, at } £. 2 - 3 - 5 a piece?
6 times 12, or 12 times 6 is 72, by 12

Price of	12		26 - 1 - 0
Multiply by	6		6
Price of	72	£.	156 - 6 - 0 *Ans.*

15. What cost 108 quartern loaves, at } £. 0 - 0 - $5\frac{3}{4}$ a piece?
12 times 9 is 108, multiply by 12

Price of	12		0 - 5 - 9%
	9		9
Price of	108	£.	2 - 11 - 9%

Note 1. I have here indulged you with cyphers, because I would do every thing that is easy to your understanding; but really, *Tyro*, it is better without them. Set down now 5 pence 3 farthings any where,

$5\frac{3}{4}$
12

- 5 - 9
9

£. 2 - 11 · 9 *Answer*

Note

Note 2. This is the only way, if you have a mind to do things quick, and off hand; for tho' there be a form in some schools to the contrary, and you may hitherto have been instructed in that way; yet, as I observed before, when you come to do a sum, you are not bound to set it down in the very form you were taught at school; for while you are doing *that*, another will answer the question over and over.

Tyro. I understand you, sir, very well, and thank you for your care; but it is only use, as you observed; for I think one full as easy as the other, and I grant it is much better for the quick dispatching of business.

16. What comes the cloathing of 144 soldiers to, at		£. 3 - 12 each man?
12 times 12 is 144, multiply by		12
12 cost		43 - 4
12	again by	12
144 cost		£. 518 - 8 *Ans.*

Tyro. You need not give me any more examples: but pray, suppose the quantity, or given number, be such as does not fall under, or cannot be found in the *Multiplication-table*, such as 17, 26, 38, 59, or the like; how then?

Philo. This is as easy as the other, as appears by the following rule.

4. *Of Numbers that are not to be found in the* Multiplication-table, *and how to work by them.*

Rule. When the given number does not fall under the common course of the table, such as 17, 38, 59, or the like; then take any two figures that will come the nearest to the number (but not exceeding it) and

work with them as in the former examples, and then add the price to that product as many times as there are odd numbers, and the work is done.

We will give you examples of 17, 38, and 59.

17. What coſt 17 groſs of corks, at £. - 10 - 6½ a groſs?

4 times 4 is 16, and 1 is 17. Mult. by		4
Price of 4 is		2 - 2 - 2
	4	4
Price of 16 is		8 - 8 - 8
Add the price of 1 groſs, *viz.*		10 - 6½
Price of 17 is		£. 8 - 19 - 2½ *Anſ.*

18. What coſt 38 loads of hay, at £. 1 - 11 - 6 a load?

6 times 6 is 36, and 2 is 38. Mult. by		6
Price of 6 is		9 - 9 -
	6	6
Price of 36 is		56 - 14 -
Mul. the price 1*l.* 11*s.* 6 by 2 gives		3 - 3 - add
Price of 38 is		£. 59 - 17 - *Anſ.*

19. What is the value of 59 ports £. 1 - 16 - each

7 times 8 is 56, and 3 is 59. Mult. by		7
Value of 7 is		12 - 12 -
	8	by 8
Value of 56 is		100 - 16 -
Then multiply 1*l.* 16*s.* by 3 is		5 - 8 - add
Value of 59		£. 106 - 4 -

This

This may be done another way.

As you ſaid before, 7 times 8 is 56, and 3 added make 59. So now you may ſay, 6 times 10 is 60; and then take, or ſubtract the value of 1 piece out of that product, and the anſwer will be the ſame as 59.

1 port is		£. 1 - 16 -	
10 times 6 is 60, firſt	by	6	
Value of 6 is .		10 - 16	
10		10	
Value of 60 is		108 - -	
Subtract 1 piece, *viz.*		1 - 16 -	ſubtract
59	*Anſ.*	£. 106 - 4 -	as above

Note 1. As ſome boys learn ſooner by one method than another, ſome maſters find it eaſier to teach them to calculate by even tens, and add the odd numbers after. Thus if it were 43, then multiply by 10, then by 4, gives 40, and then add the 3 odd numbers. So 64 is $10 \times 6 \times 5 = 65$, and thus for any numbers.

Note 2. If you are perfect in what has already been ſhewn you, it will be very eaſy to perform any ſum in common arithmetic, relating to buſineſs, not only within the compaſs of the table, but in much larger numbers. As for inſtance; ſuppoſe it was required to tell the value of 1000 moidores: or what will it coſt to cloath 1000 men, at 1*l.* 7*s.* a man? Here 10 times 10 make 100, and 10 times 100 make 1000. Multiply therefore 1*l.* 7*s.* by 10; then that product by 10; and this laſt product by 10 will give the anſwer, *viz.* £. 1350.

QUESTIONS *undone for practice.*

1. What comes 33 dozen of candles to, at 4*s.* 9*d.* a dozen? *Answer* £. 7 - 16 - 9. Multiply by 3, and then by 11.

2. How much is 46 pistoles, at 17*s.* 6*d.* each? or what cost 46 pieces of *Irish*, at 17*s.* 6*d.* a piece. *Answer* £. 40 - 5. Multiply by 5, then by 9, and add 17*s.* 6*d.* more for the odd 1.

3. What cost 63 barrels, at £. 1 - 5 - 9 a barrel? *Ans.* £. 81 - 2 - 3.

4. What comes 49 months salary to, at 3 guineas a month? *Ans.* £. 154 - 7.

5. What comes 105 gallons of rum to, at 7*s.* 6*d.* ½ a gallon? *Ans.* £. 39 - 11 - 10 ½. 10 times 10 make 100, and then multiply 7*s* 6*d.* ½ by 5, for the odd 5 gallons, and add it to the last product.

6. If I spend 1*d.* ¾, or 7 farthings a day, how much is that a year, allowing 365 days? *Answer* £. 2 - 13 - 2¾

First, 10 times 10 make 100; then multiply the answer of 100 by 3, it gives you the expences of 300 Days, *viz.* 2*l.* 3*s* 9*d.* Now there are but 65 days behind; therefore set down 7 farthings in another place, and multiply it by 8, and that product multiplied by 8 again makes 64, and then add once 1*d.* ¾ for the odd day, gives 9*s.* 3*d.* ¾ for the 65 days, which added to the 300 days, gives £. 2 - 13 - 2¾. *Secondly*, or thus, which is shorter, 11 times 11 is 121; then multiply the product of 121 by 3, give 363, and then add 2 times 1*d.*¾, *viz.* 3*d.* ½ to that product, gives £. 2 - 13 - 2¾, as before.

As I have added the answers to these questions, I hope, *Tyro*, you will try to prove them at large: for if you mind and make yourself master of this rule, you will be able to cast up any common thing, as quick, or quicker than by any other method, and with far greater ease.

As

As for *cross multiplication*, *viz.* multiplying money by money, or feet and inches by feet and inches, it is better let alone for the present; therefore, we will proceed to *division*.

DIALOGUE V.

SECTION I.

Of DIVISION.

Tyro. WHAT is *division*, and what does it teach?

Philo. *Division* is just the reverse of *multiplication*; for, as any sum is encreased as many times as the figure you multiply by; so in *division*, the number is decreased, or divided into as many parts, as the value of the figures you divide by.

Tyro. What is to be learned, or observed in this rule?

Philo. There are four things very necessary to be known in *division*, *viz.*

First, The *Dividend*, which is the sum given to be divided.

Secondly, The *Divisor*, or the number you are to divide by.

Thirdly, The *Quotient*, or *answer*, which shews you how many times the *Divisor* is contained in the *Dividend*: or into how many parts the *Dividend* is divided.

Fourthly, The *Remainder*, which is a fractional part of the *Quotient*: but this does not concern you as yet.

Tyro. Am I to get this by heart?

Philo. You are not bound to get it just word for word; but you should understand well what the first 3 mean.

Tyro.

Tyro. Be so kind, sir, as to explain them a little more for me.

Philo. Observe then: let us take any two numbers suppose 24, and 6: now 24 is the *Dividend*, and 6 is the *Divisor*. Then I ask how many times 6 can I have in 24, and the answer is 4; which 4 is called the *Quotient*. So also, suppose it was required to divide 108 by 12 or into 12 parts, then every part will be 9; for 12 times 9 is 108. Now 108 is the *Dividend* or sum to be divided; 12 is the *Divisor*, or what you divide by, and 8 is the *Quotient*.

Tyro. Now, sir, I apprehend you quite well.

Philo. Then I will proceed to some examples in single figures; but pray observe the following rule.

1. *Of dividing by single figures in one line only.*

Rule. First ask how many times the *Divisor* is contained in the first figure of the *Dividend*, and if the *Divisor* be larger than the first figure in the *Dividend*, then seek how many times you can have it in the first two figures in the *Dividend*, and set the figure down accordingly: and if any thing remains from the first figure in the *Dividend*, carry it to the second; and if any thing remains in the second figure, carry it to the third; remembering always in this short *Division*, that if one remains you call it 10, if 2 remains you call it 20, if 5 50, and so on, carrying the remainder of one figure to another in your mind.

Note, To make the work both shorter and easier, remember that 2's is read *two's*, 3's is *three's*, 5's is *five's*, 12's is *twelve's*, and so for any other figure: thus, the 7's in 14, is read thus, the *sevens* in 14; and the 6's in 24, is the *sixes* in 24; which is the same as if I should ask, how many *sixes* can I have in 24, but only shorter and more convenient.

An example or two will, with care, make it familiar to you.

Ex-

Ex. 1.	*Dividend*	Ex. 2.	*Dividend*
Divisor	3)39	*Divisor*	4)168
Quotient	13 *Ans.*	*Quotient*	42 *Ans.*
	3		4
Proof	39	*Proof*	168

Now observe: In *ex.* 1, I ask how many *three's* I can have in 3; or I say the 3's in 3 is once; therefore I set down one under the 3 in the *dividend*, and as there remains nothing over, I ask how many 3's I can have in 9; or say, the 3's in 9 is 3 times 3, and nothing over; therefore, I set down 3 under the 9, and it is done. Again,

In *ex.* 2, I say the 4's in 1 I can't have; but taking the next figure to it, *viz.* 6, I say the 4's in 16 is just 4 times; therefore, I set down 4 under the 6 in the *dividend*, and as there is nothing remains, I only say, the 4's in 8 is twice, therefore I set down 2 under the 8, and it is done. Now to prove it I multiply 42, the *quotient*, by 4, the *divisor*, and find it 168 like the *dividend*.

Ex. 3.	*Dividend*	Ex. 4.	*Dividend*
Divisor	8)37192	*Divisor*	9)245052
Answer	4649	*Answer*	27228
	8		9
Proof	37192	*Proof*	245052

Now observe, *Tyro*, in *ex.* 3, I divide by 8, saying, the 8's in 3 I can't; but the 8's in 37 is 4 times 8 is 32, and 5 over; I therefore set down 4 under the 7, and carry 5 to the next figure, which is 1, which I now call

51; (for what you carry from one figure you muſt always place before the next figure) then I ſay the 8's in 51 is 6 times 8 is 48, and 3 over; which 3 I now carry to the 9, and it is 39; therefore, I ſay the 8's in 39 is 4 times 8 is 32, and 7 over; this 7 I now place before the 2, and it is 72; then I ſay the 8's in 72 is juſt 9 times, and the work is done. *To prove it*, I multiply the *quotient* or anſwer by the *diviſor* 8, and find the *product* the ſame as the *dividend*. In *Ex.* 4, I ſay, the 9's in 2 I can't; but the 9's in 24 is 2 times, or *twice* 9 is 18, and 6 over, which 2 I place under the 4, and carry 6 to the next figure, which is 5, and call it 65; then I ſay the 9's in 65 is 7 times 9 is 63, and 2 over, which 2 I place before the cypher (0) and it is 20; then I ſay the 9's in 20 is *twice* 9 is 18, and 2 over, which I carry to the 5, and it is 25; then I ſay the 9's in 25 is *twice* 9 is 18, and 7 over, which I ſet before the laſt figure 2, and it is 72; then I ſay the 9's in 72 is juſt 8 times. *To prove it*, I multiply the *anſwer* by 9, and the *product* will be the ſame as the *dividend*.

EXAMPLE 5.		EXAMPLE 6.	
Divide by	7)417296	*Divide by*	9)9471636
Anſwer	59613-5*	*Anſwer*	1052404
	7		9
Proof	417296	*Proof*	9471636

* Here in *example* 5, there is 5 remains at laſt; therefore, I ſet it at the end of the anſwer, parting it with a ſtroke, thus, -5; and when I prove the work I multiply by 7, and take the remainder 5 in, ſaying, 7 times 3 is 21, and 5 is 26, 6 and I carry 2, &c.

Ex-

EXAMPLE 7.		EXAMPLE 8.	
	8)716206		5)12631908
Answer	89525-6	*Answer*	2526381-3
	8		5
Proof	716206	*Proof*	12631908

More examples for practice.

7)697220716 9)407217943

Tyro. I understand what you have shewn me very well; but pray, before you come to long *division*, give me an example at large to divide by *twelve* the short way.

Philo. I will: let it be required to divide 1479908 by 12. I set it down thus:

EXAMPLE 11.	EXAMPLE 12.
12)1479908	12)59904965
123325-8	4992080-5

Here I divide by 12, saying, the 12's in 14 is *once* 12, and 2 over, and this 2 I carry to the 7 and it is 27; then I say the 12's in 27 is *twice* 12 is 24, and 3 over, that is 39; then I say the 12's in 39 is 3 times 12 is 36, and 3 over, that is 39 again; therefore, I set down another 3, and carry the 3 that is over to the next figure, which is a cypher, calling it 30; then I say the 12's in 30 is *twice* 12 is 24, and 6 over, which is 68; now the 12's in 68 is 5 times 12 is 60, and 8 over, which I place after the sum, thus, -8, and it is done.

In *Ex.* 12, I say the 12's in 5 I can't; but the 12's in 59 is 4 times 12 is 48, and 11 over, which 11 carried to, or joined to the next figure 9 is 119; then I say the 12's in

in 119 is 9 times 12 is 108, and 11 over again, which I now carry to, and join to the next figure 0, and call it 110; then say the 12's in 110 is 9 times, and 2 over, which 2 I join to, or carry to the 4, and it is 24: now the 12's in 24 is just *twice*, and nothing over; therefore I now begin a-fresh at the 9, saying, the 12's in 9 I can't, but the 12's in 96 is just 8 times: *lastly*, I say, the 12's in 5 I can't have; therefore, I set down a Cypher under the last figure 5, and drawing a short line, set the 5 that remains after it, thus, -5.

Now *to prove it*, I multiply back by 12, and take in the 5, saying, 12 times 0 is 0, but 5 is 5; then 12 times 8 is 96, 6 and I carry 9, *&c.* till I go through the whole.

☞ *Note*, *Multiplication* is an infallible proof for *division*; for if you multiply the *quotient* by the *divisor*, you will have the same figures as are in the *dividend*, always remembering to take in the remainder with the first figure you begin to multiply.

Tyro. I humbly thank you, sir; I am quite satisfied with what you have shewn me; and now, if you please, I shall be obliged to you to shew me how to divide by several figures.

Philo. You will soon learn it with care: you must note there are 3 or 4 ways to work *division*; but as my intent is not for curiosity, but improvement, I shall only shew you that method which is most *natural* and *practicable*; you may at any time learn the rest.

2. *Of dividing by 2, 3, or more figures.*

Rule. First *seek*, or ask how many times the figures in the *divisor* are contained in the same number of figures in the *dividend*, and put that figure in the *quotient*. *Secondly*, *multiply* now the *divisor* by the said figure in the *quotient*, and place it under those figures in the *dividend* that you began to work with, always observing, that the *product* be not larger than the figures

gures in the *dividend*; for if they are, you must rub out, or cancel the figure in the *quotient*, and put one of a less denomination. *Thirdly*, this being done, *subtract* now the *product* from that part of the *dividend* it stands under, and to the remainder bring down the next figure in the *dividend*, placing, or joining it to the last figure of the *remainder*. Then *seek* how many times the *divisor* is contained in these figures; then multiply the *divisor* by the said figure; then *subtract* again; and *lastly*, bring down the next figure in the *dividend*, as before; and thus proceed till you have no more figures in the *dividend* to bring down, and the work is done.

☞ *Note* 1. Every time you *subtract*, observe whether the *remainder* be larger than the *divisor*; for if it be, you must put a larger figure in the *quotient*.

Note 2. Whenever you take a figure down from the *dividend*, and join it to the *remainder*, and that is still less than the *divisor*; then always put a cypher in the *quotient*, and bring down another figure of the *dividend*.

An example or two at large will make it easier.

Example I.

```
          Dividend
Divisor 15)19260(1284 quotient, or answer
           15
           --                 Quotient 1284
            42                multiply by 15 divisor
            30                         ----
            --                         6420
            126                       1284
            120                       -----
            ---               Product 19260 dividend
              60
              60
              --
               0
```

Now

Now obſerve: having 2 figures in the *diviſor*, I aſk how many times they are contained in the 2 firſt figures of the *dividend*, (*viz.* 19) and find it *once*; therefore I ſet 1 in the *quotient*. *Secondly*, I multiply the *diviſor* 15 by 1, ſaying, *once* 5 is 5, *once* 1 is 1, and place it under the 19. Then, *thirdly*, *ſubtract* 15 from 19, and there remains 4; and *laſtly*, I bring down the next figure in the *dividend* (*viz.* the 2) and join it by the ſide of the *remainder* 4, and it is 42; and now I begin as at the firſt, and ask how many times 15 are contained in 42, and find it *twice*; therefore I ſet down 2 in the *quotient*: then I multiply the *diviſor* 15 by 2, which is 30, and place it under 42; this done, I *ſubtract* 30 from 42, and there remains 12; and then again, I bring down the next figure of the *dividend* (*viz.* 6) and place it by the ſide of 12, and it is 126. Then I aſk how many times 15 I can have in 126, and find it 8 times; therefore, I put an 8 in the *quotient*, and multiply 15 by it, which is 120, which I place under 126. Then I *ſubtract* 120 from 126, which is 6, and bring down the laſt figure in the *dividend*, which is a cypher (0) and it makes 60; then I ſeek how many times 15 I can have in 60, and find it 4 times; then I multiply 15 by 4, and find it juſt 60, which I place under the other 60, and the work is done.

RULE 2.

Note 1. When there are ſeveral figures in the *diviſor*, it is eaſier for a learner to aſk how many times the firſt figure of the *diviſor* is contained in the firſt figure of the *dividend*, and place the times in the *quotient*; then multiply the whole *diviſor* by the *quotient* figure, and if the product be more than the figures which belong to the *dividend*, you muſt make trial of a leſs figure, and put it in the *quotient*.

Note

Note 2. If the firſt figure of the *diviſor* be larger than the figure in the *dividend*, then take 2 figures in the *dividend*, and ſeek how many times the firſt figure of the *diviſor* is contained in them.

Note 3. You muſt remember, that in making trial how often the firſt figure in the *diviſor* is contained in two figures of the *dividend*, it will ſometimes appear to be 10, or 12 times; but obſerve, it never can be above 9 at moſt, and oftentimes not ſo many as it appears to be.

The ſame example further demonſtrated.

Let it be required to divide 19260 by 15?

Firſt, I ſet the *dividend* down on a ſlate, and make a couple of crooked lines at the ends of it, in the firſt of which I place the *diviſor*, thus, 15)19260(and the other is to place the *quotient* in.

Secondly, I aſk how many times the firſt figure of the *diviſor* is contained in the firſt figure of the *dividend* and find it *once*, therefore, I place a 1 in the *quotient*, and multiply the whole *diviſor* by it, and place the product under the two firſt figures of *the dividend*, and ſubtract it therefrom, and it will ſtand thus:

Second work.

```
          15)19260(1
             15
             ——
*Remains*    42
```

Thirdly, to this remainder 4 I bring down the next figure, *viz.* 2, (always making a dot under the figure I bring down) and it is 42; then I aſk how often the firſt figure 1 in the *diviſor* is contained in 4, and it is 4 times; but upon trial, I find 4 times 15 is 60, therefore,

therefore, as 60 is more than 42, I muſt take a leſs figure; I therefore make trial of a 2, and find twice 15 is 30; which I place under 42, and ſubtract it, and there remains 12, which ſtands thus:

Third work.

```
15)19260(12
   15··
   ——
    42
    30
   ——
Remains  126
```

Fourthly, I now make a dot under the 6, and bring it down by the ſide of the 12, and it is 126; then I aſk how many of the firſt figures of the *diviſor* I can have in the firſt two figures of the *dividend*, and find it 12 times; but according to *note* 3, it never goes above 9 times; therefore, I multiply 15 by 9, and it is 135: now I can't take 135 out of 126, therefore 9 times is too much and I make trial of a leſs figure, to wit, 8, which I put in the *quotient*, and multiply 15 by it, which is 120, and place it under 126, and there remains 6, which ſtands thus:

Fourth work.

```
15)19260(128
   15···
   ——
    42
    30
   ——
    126
    120
   ——
Remains  6
```

Laſtly,

Laſtly, I make a dot under the cypher (o) and bring it down by the ſide of the 6, thus, 60; then I aſk how many times the firſt figure of the *diviſor* is contained in 6, or the *ones* in 6 is 6 times; but upon trial, I find it will go but 4 times; therefore, I place a 4 in the *quotient*, and multiplying 15 by it, find it to be 60, which I place under the other 60, and there remains 0, and the work is done, as under.

Laſt work.

```
Diviſor 15)19260(1284 quotient
           15...                 Proof
           ----                  1284 quotient
            42                     15 diviſor
            30                   ----
           ----                  6420
           126                  1284
           120                  -----
           ----        Product  19260 dividend
             60
             60
           ----
              0
```

☞ *Note*, the reaſon of making dots under every figure you take down, is becauſe you may not miſtake which figure comes next in courſe; but when you are quite perfect you need not trouble yourſelf with them.

Tyro. This is a plain demonſtration, and I ſee now the manner of working *diviſion*, though, I own, I am not perfect yet.

Philo. I ſhall give you an example or two more then, and ſhew you at the ſame time how to prove it by *addition*.

Example 2.

Divide by 375)24172196(64459 *Answer*		
	*2250	
	1672	
	*1500	64459
		375
	1721	
	*1500	322295
		451213
	2219	193377
	*1875	
		24172125
	3446	71 add *rem.*
	*3375	
		24172196 *proof*
Remains	*71	
Proof	24172196	

The PROOF.

This example is proved the ſame as the laſt, by multiplying the *quotient* by the *diviſor*, and afterwards adding the *remaind r* to it.

PROOF *by* Addition.

Though *multiplication* be an infallible proof of *diviſion*, yet in large ſums, *addition* is much ſhorter and eaſier, and when the work is right, is as infallible as the other. If indeed it be objected, that a boy may *vamp*, or alter the figures (as in proving *multiplication* by the croſs) in order to make the ſum come right, there is the ſame reaſon to ſuppoſe (without his maſter's inſpection) that he may alter his *multiplication* ſum, when he comes to add it up, that it may fall right with the *di-*

dividend. Besides, as in the very work of every *division* sum, the *divisor* is naturally multiplied into every figure of the *quotient*, what need it to be done again? And I persuade myself, that those masters, that have made use of this method of proving *division*, will acknowledge, that it is not only much easier to the scholar, but also much more improving; because he must set his figures in order, right under one another, or else he will be puzzled to add the sum up: he cannot therefore avoid improving himself (in some measure) by this method.

To prove Division *by* Addition.

Rule. Add the Remainder of all the different products of the *divisor* together, in order as they stand, and their sum will be the same as the *dividend.* Or thus: add the remainder (if any) and all the lower lines together in order, as they stand, and you will find their sum the same as the *dividend*, if the work be right.

Note, in *example* 2, I have made stars (thus *) against those figures which are to be added together; which you see is the lower row of the two; for the top row is never added.

Two

Two examples with their proofs.

EXAMPLE 3.

```
Divide by 5267)821695074(156008 answer
                *5267 ....
                 -----------
                 29499
                *26335
                 -----------
                  31645
                 *31602
                 -----------
                    43074
                   *42136
                 -----------
Remains               *938
                 -----------
Proof           821695074
```

EXAMPLES 4.

```
Divide by 9758)94076257(9640 answer
                *87822...
                 ---------
                  62542
                 *58548
                 ---------
                   39945
                  *39032
                 ---------
Remains            *9137
                 ---------
Proof           94076257
```

Here in the first example, I take all the lower lines of the two, against which this mark (*) is placed, and beginning at the remainder, I say 8 and 6 is 14, 4 and I carry

I carry 1, to 3 is 4, and 3 is 7; then 9 and 1 is 10, that is 0, and I carry 1 to 2 is 3, and 2 is 5; then 4 and 5 is 9, then 6 and 3 is 9, and 7 is 16. 6 and I carry 1, to 1 is 2, and 3 is 5, and 6 is 11, that is 1, and I carry 1; to 3 is 4, and 6 is 10, and 2 is 12, that is 2, and carry 1, to 2 is 3, and 5 is 8. And thus you may proceed to prove the other example.

Tyro. I confeſs I could not have thought it had been ſo eaſy; and it is (in effect) proving it by Multiplication, as you obſerved.

Philo. I am glad you underſtand it; and I hope you will take ſuch care as to have no occaſion to prove the work at all. Perſons in buſineſs, *Tyro*, cannot go thro' theſe forms; if they ſuſpect they have done wrong, they look over the work a ſecond time, and that is a ſufficient proof in general.

Of CONTRACTIONS.

Tyro. What is the uſe of contractions?

Philo. When the diviſor conſiſts of ſeveral cyphers after a figure, or figures, then cut them all off, or ſeparate them from the figures with a daſh of your pen or pencil; and alſo remember at the ſame time to cut off as many cyphers or figures in the dividend; then work the ſum as if ſuch cyphers never had been there at all, and you will have the ſame anſwer.

Ex. 1.	Ex. 2.	Ex. 3.
1|000)9250|000	4|00)5476|00	12|00)54762|00
9250 *Anſ.*	1369 *Anſ.*	*Anſ.* 4563—6

Here cut off all the cyphers in the diviſor, and as many cyphers in the dividend, and divide only by the ſingle figures, and if any thing remains, I ſet it after it. As in example 3, there is 6 remains, which I ſet after the anſwer, thus,—6.

Tyro. This is easy enough; and now, if you please, I should be glad to know what you mean by *Division of Parts*.

SECTION I.

DIVISION *of Parts*.

Philo. DIVISION *of Parts* is the dividing by any two single Figures in their Parts, which 2 Figures multiplied together will be equal to the *Divisor*.

Tyro. I do not rightly apprehend you.

Philo. You remember, *Tyro*, in *Questions* of *Multiplication*, that when any number was given in the table, you found two such figures, which, when multiplied together, would make that number; so here also you do the same, only with this difference, that you here divide by them instead of multiplying. Thus, suppose I was to divide by 24, by 32, by 48, or by 72, I first divide by 3, then by 8; for 3 times 8 is 24, and the last Quotient is the answer. So if I divide by 48, I divide that number first by 8, and then the *Quotient* I divide by 6, and have the proper answer, &c.

EXAMPLE I.

Divide 16488 *by* 24. Here 6 times 4 is 24.

First by 6) 2748 *Quotient by* 6

Then by 4) 687 *Quotient by* 24

Ex-

EXAMPLE 2.

Divide 126216 *by* 72. 6 times 12 is 72.

First by 6) 21036 *Quotient by* 6

Then by 12) 1753 *Quotient by* 72

Tyro. I understand you; this is only then the reverse of *Multiplication.*

Philo. Nothing more, as you may more plainly see by what follows.

2. *Of dividing* MONEY.

Division of Money is just the reverse of *Multiplication of Money*; for as in *Questions* of *Multiplication*, the money is encreased somany times, according to the given number; so here the money is decreased, or divided into as many parts as the given number is.

It will be easier to your comprehension to take an example in *Multiplication*, and prove it by *Division.*

If 1 ell cost £. 1 - 3 - 8 What cost 40 ells?

		8
8		9 - 9 - 4
5		5
40	£.	47 - 6 - 8 *Ans.*

PROOF *by* DIVISION.

1. If 40 ells cost £. 47 - 6 - 8, what cost 1 ell?

Here I divide back by the same figures I multiplied by, carrying the pounds to the shillings, and the remainder of the shillings to the pence, doing by *twenties*, and by *twelves*, as in *Addition.*

by 5)£47 - 6 - 8

Then by 8) 9 - 9 - 4 Price of 8

£.1 - 3 - 8 Price of 1 *Anſ.*

Here I divide firſt by 5, ſaying, the 5's in 47 is 9 times 5, and 2 over, which is £.2; this £.2 I carry to the 6 *s.* and it is £. 2 - 6, or 46 *s.* then I ſay the 5's in 46 is 9 times 5 is 45, and 1 over, that is 1 ſhilling, which I carry to the 8 pence, and it is 1 *s.* 8 *d.* or 20 pence, then I ſay the 5's in 20 is 4 pence. Now I divide this £.9 - 9 - 4 by 8, ſaying, the 8's in 9 is *once*, and £. 1 over, which I carry to the 9 ſhillings, and it is £. 1 - 9, or 29 ſhillings; then I ſay, the 8's in 29 is 3 times 8 is 24. and 5 ſhillings over, which I carry to the pence, and it is 5 *s.* 4 *d*, or 64 pence; then I ſay the 8's in 64 is 8 times. So is the price of 1 ell £. 1 - 3 - 8; as appears alſo by the work of *Multiplication.*

Example 2.

Suppoſe £.88 - 4 - 0 be divided among 56 poor widows, how much is the ſhare of each?

by 7)£88 - 4 - 0 Here 7 times 8 is 56.

by 8)12 - 12 - 0 Share of 8

Anſ. 1 - 11 - 6 Share of each.

If you underſtand the nature of multiplying any ſum by any figure, you ſee you may as readily divide it back again.

Tyro. Sir, I ſee the way of working it very plainly. Have you any thing farther to add.

Philo. Nothing more, as you ſay you are perfect; only ſome queſtions for to exerciſe you.

3. Questions

3. QUESTIONS *to exercise* DIVISION.

1. Divide 822306485 by 1715. *Ans.* 479479.
2. Divide 150348045 by 285. *Ans.* 527537.
3. Divide 47198714 by 6357. *Ans.* 7425, and 4346 remains.

QUESTIONS *in* MONEY.

1. Divide 1 *l.* 16 *s.* 8 *d.* into 5 parts. *Ans.* 7 *s.* 4 *d.*
2. Divide 4 *l.* 8 *s.* 1 *d.* ½ into 9 parts. *Ans.* 9 *s.* 9 *d.* ½.
3. Divide 25 *l.* 5 *s.* 4 *d.* into 32 parts. *Ans.* 15 *s.* 9 *d* ½.
4. A gentleman left by will 100 pounds among 3 ſcore (or 60) poor perſons, to be paid every *Chriſtmas-Day*; how much is each to receive?

I leave this laſt queſtion wholly for the learner to do. And now, *Tyro*, you are come to that rule, in which you will exerciſe all the other *four*; therefore, if you be not perfect, you will be at a loſs very often.

DIALOGUE VI.

SECTION I.

Of REDUCTION.

Tyro. WHAT is *Reduction*, and what does it teach?

Philo. *Reduction* is a rule compounded of all the foregoing rules; being a proper exerciſe for the better perfecting you in them. It teaches to *reduce* things of one denomination into another, and is of excellent uſe in common affairs of life.

Tyro. How many parts is this rule divided into?

Philo. Two. 1. *Reduction ascending.* 2. *Reduction descending.*

Tyro. What do you mean by *Reduction ascending.*

Philo. *Reduction ascending* is when things of a *small* are *reduced,* or *brought* into things of a *greater* denomination; as farthings into pence, pence into shillings, shillings into pounds, or ounces into hundred weights: And *this* remember is done by *Division* only, by dividing by as many of the *less,* as make one of the *greater* denomination.

Tyro. What do you mean by *Reduction descending.*

Philo. *Descending* is just the reverse of the other; for by *Reduction descending,* things of a *great* are *reduced* or *brought* into things of a *less* denomination; as pounds into shillings, shillings into pence or farthings; *Cwts.* into pounds, tons into quarters, miles into yards, yards into inches, and the like; and *this* remember is always done by *Multiplication* only, by multiplying the *greater* denomination by as many as it contains of the *less* denomination.

Tyro. Then I find *Reduction descending* is the easier of the two, as it is performed by *Multiplication* only.

Philo. It is so: And that is the reason it is commonly taught first.

Note, I shall prove every sum of *Reduction descending* by *Reduction ascending,* and then you will see the nature of both.

1. *Of* REDUCTION descending.

Tyro. How do you say this is performed?

Philo. By this rule. *Multiply every denomination by as many as the next less denomination contains, and you have the answer, which is called* Reduction descending. Then to prove the work, *divide back by the same figures you multiplied by, and this is called* ascending.

A few questions will make it plain to you.

Quest.

Quest. 1. In 12 pounds how many shillings?

£. 12
Multiply by 20 Shillings in a £.

Ans. 240 Shillings

PROOF *ascending.*

In 240 shillings how many £. *sterling?*
Here I divide back by 20, as before I multiplied.

2|0)24|0 Shillings

£. 12 *Ans.*

Here I cut off the cyphers in the *Divisor* and *Dividend* with a dash of my pen, and divide by 2 only, and the answer is 12.

Quest. 2. In £. 23 - 14 *s.* how many shillings and pence.

£. 23 - 14
First by 20 and take in the 14.

474
Then by 12

5688 Pence.

PROOF.

In 5688 pence how many shillings and £'s *sterling.*
Divide by 12)5688 Pence

Then by 2|0)47|4 Shillings

£. 23 - 14 as above

 Here

Here I divide by 12, and it brings the pence into ſhillings; then I divide by 20, cutting off the cypher, and cutting off the 4 alſo: Then I divide by 2 only, ſaying, the 2's in 4 is *twice*, the 2's in 7 is 3 times 2 and 1 over, which 1 I place before the figure 4, that I alſo cut off, and it is 14 ſhillings.

This is a *general rule*; obſerve, that when you cut off any figure, or figures, what remains in dividing muſt be placed before them, and is the true remainder.

Tyro. This is very plain I own.

Philo. Then we will proceed to *Queſtion* 3.

Queſt. 3. In £. 45 - 17 - $6\frac{3}{4}$ how many ſhillings, pence, and farthings?

$$\begin{array}{rl} & £.45 - 17 - 6\frac{3}{4} \\ \textit{by} & 20 \\ \hline & 917 \text{ Shillings} \\ \textit{by} & 12 \\ \hline & 11010 \text{ Pence} \\ \textit{by} & 4 \\ \hline \textit{Anſ.} & 44043 \text{ Farthings} \end{array}$$

Here I multiply £. 45 by 20, and take in the 17 ſhillings, ſaying, 0 is 0 but 7 is 7; then twice 5 is 10, and the 1 by the ſide of the 7 ſhillings is 11, that is 1, and I carry 1; then twice 4 is 8, and 1 is 9, which is 917 ſhillings. Again, I multiply the ſhillings by 12, and take in the odd ſix-pence; and *laſtly*, I multiply the pence by 4, and take in the odd 3 farthings, ſo is the work done.

PROOF

PROOF *by* Reduction ascending.

In 44043 farthings, how many pence, shillings, and pounds *sterling*?

Here I divide only back by the same figures I multiplied by, and it is done.

Divide by 4)44043 Farthings

by 12)11010 Pence, and 3 farthings over

by 2|0) 91|7 Shillings, and 6 pence over.

£. 45 - 17 - 6¾ as above.

Tyro. I understand it very well indeed.

Philo. Then I need not instruct you any more till some difficulty offers.

Quest. 4. In £99 - 19 - 11¾, how many shillings, pence, and farthings?

£. 99 - 19 - 11¾
Multiply by 20

1999 Shillings
by 12

23999 Pence
by 4

95999 Farthings

In 95999 farthings, how many pence, shillings, and pounds?

Divide by 4)95999 Farthings

by 12)23999—3

by 2|0) 199|9—11

£. 99 - 19 - 11¼

5. In 30 guineas, how many shillings and pence?

```
  30 Guineas
  21 Shillings make 1 guinea
 ---
  30
 60
 ---
 630 Shillings
  12
 ---
7560 Pence
```

PROOF.

In 7560 pence, how many shillings, and guineas?

```
12)7560 Pence
   ----
21) 630(30 Shillings Ans.
    63
    --
     00
```

And thus you see, *Tyro*, that any sum, of any name or denomination, may be reduced to another.

Quest.

Quest. 6. In £. 472 - 15 shillings, how many crowns, shillings, groats, and pence?

	£. 472 - 15	
Multiply by	4	the crowns in a £. and count the 15 *s.* for 3 crowns, saying, 4 times 2 is 8, and 3 is 11.
Crowns	1891	
	5	Shillings 1 crown
Shillings	9455	
	3	Groats 1 shilling
Groats	28265	
	4	Pence is 1 groat
Pence	113460	*Ans.*

PROOF.

In 113460 pence, how many groats, shillings, crowns and pounds?

Divide by 4)	113460	Pence
by 3)	28365	Groats
by 5)	9455	Shillings
by 4)	1891	

£. 472—3 Crowns over, *viz.* 15 shillings, as above.

Note 1. To bring pounds into pence at one operation, *multiply by* 240, because 240 pence make 1 pound.

To bring pounds into farthings at one operation, *multiply by* 960, the farthings in a pound. On the contrary.

Note

Note 2. To bring pence into pounds, *divide by* 240. To bring farthings into pounds, *divide by* 960

AVOIRDUPOISE-WEIGHT.

Quest. 7. In 17 *C.* 3 *qrs.* 15 *lbs.* how many *Cwt. qrs.* and *lbs.*

```
      C.  qr.  lb.
      17 - 3 - 15
by     4 qrs. make 1 Cwt.
    ------
      71 qrs.
by    28 lb. 1 qr.
    ------
     573
    143
    ------
    2003 lbs. Ans.
```

Here I multiply by 4, and take in the odd 3 quarters; then I multiply by 28, and take in the odd 15 pounds, and the work is done.

PROOF.

In 2003 pounds weight, how many quarters of hundreds, and hundred weight?

```
28)2003(71 qrs.          qrs. lb.
   196                4)71 - 15
   ----                ----------
    43                 17 - 3 - 15 as before
    28                 C.   qr.  lb.
   ----
    15 lb. over
```

Quest.

Quest. 8. In 17 *C.* 3 *qrs.* 15 *lb.* how many *qrs. lbs.* and *oz.*

```
    C.  qrs.  lb.
    17 - 3 - 15
by   4
  ------
    71 qrs.
    28
  ------
   573
   143
  ------
  2003 lb. *
    16
  ------
 12018
 2003
  ------
 32048 oz. Ans.
```

Note, The following method is very useful in many cases to reduce hundred weights into pounds, being both short and expeditious, *viz.*

* Set down the hundreds 4 times under one another in the following manner, and add the odd pounds besides, gives the answer.

```
17 Cwt.
 17
17
17
 99 lb. in 3 qrs. 15 lb.
------
2003 lb. *
```

PROOF.

In 32048 ounces, how many *lbs. qrs.* and *Cwt.*
Divide this back again by 16, 28, and 4, and you will have 17 *C.* 3 *qrs.* 15 *lb.*

TROY.

TROY-WEIGHT.

Quest. 9. In 5 *lb.* 2 *oz.* 14 *dwts.* of silver, how many *oz. dwt.* and *grs.*

	lb. oz. dwts.	
	5 - 2 - 14	
12 *oz.* 1 *lb. Troy by*	12	and take in 2 *oz.*
	62	*ozs.*
20 *dwts.* 1 *oz. by*	20	and take in 14 *dwts.*
	1254	*dwts.*
24 *grs.* 1 *dwt. by*	24	
	5016	
	2508	
	30096	Grains *Ans.*

PROOF.

Divide back by the same figures, 24, 20, and 12, and have a regard to the remainder, you'll find it 5 *lb.* 2 *oz.* 14 *dwts.* Do you now understand what has been shewn you?

Tyro. I do very well; and I can see that *Reduction ascending* is only the proof to the other, and that they prove one another in every respect.

Philo. If you are perfect in what I have shewn you, there is no occasion to run through all the weights and measures, for they are done after the same manner.

Tyro. I know it, Sir; but every example is a fresh encouragement.

Philo. True; and I am as ready to give them.

DRY-

DRY-MEASURE.

Quest. 10. In 14 loads, 15 bushels, how many bushels, and pecks.

	L. *B.*	
	14 - 15	
40 bushels 1 load, *mult. by*	40	and take in the 15 bush.
	575	
	4	Pecks 1 bushel
	2300	Pecks *Ans.*

PROOF.

In 2300 pecks, how many bushels and loads?

Divide 2300 by 4, and that quotient divide by 40, and 15 will remain; which is 14 loads, 15 bushels.

LIQUID-MEASURE.

Quest. 11. In 12 hogsheads of wine how many gallons and pints?

	12	Hogsheads
by	63	Gallons in 1 hogshead of wine
	36	
	72	
	756	Gallons
by	8	Pints 1 gallon
	6048	*Ans.*

PROOF

PROOF.

In 6048 pints how many gallons and hogsheads?

Divide this back again by 8, and by 63, and you will have just 12 hogsheads.

BEER-MEASURE.

Quest. 12. In 18 butts of beer how many hogsheads and gallons?

	18	Butts
by	2	Hogsheads 1 butt.
	36	Hhds.
by	54	Gallons 1 hogshead of beer.
	144	
	180	
	1944	Gallons *Ans.*

PROOF.

In 1944 gallons of beer, how many hogsheads and butts? *Ans.* 18 butts.

CLOTH-MEASURE.

Quest. 13. In 26 yards, 3 quarters, how many quarters and nails?

	Yards qrs.	
	26 - 3	
by	4	quarters 1 yard, and take in the 3 *qrs.*
	107	*qrs.*
by	4	nails 1 *qr.*
	428	Nails.

PROOF.

PROOF.

In 428 nails, how many quarters and yards?
Divide 428 by 4, and then by 4 again. *Anſ.* 26 yards, 3 qrs.

Note, This, and the two following queſtions, are of great ſervice in the *Rule of Three direct*; where it is often required to reduce yards and ells into quarters, and the contrary.

Queſt. 14. In 26 ells *Engliſh*, 3 quarters, how many quarters and nails?

	Ells qrs.
	26 - 3
by 5, and take in 3 qrs.	5 qrs. 1 ell *Engliſh*.
	133 qrs.
	by 4 Nails 1 qr.
	532 Nails

PROOF.

In 532 nails, how many quarters and ells *Engliſh*?
Divide 532 by 4, and by 5, you'll have 26 ells *Engliſh*, 3 qrs.

Queſt. 15. In 26 ells *Flemiſh*, 2 quarters, how many quarters and nails?

Ells Fl. qrs.
26 - 2
by 3 qrs. 1 ell *Flemiſh*, take in 2 qrs.

80 qrs.
by 4 Nails 1 qr.

320 Nails. *Anſ.*

PROOF.

PROOF.

In 320 nails, how many quarters, and ells *Flemish*?
Divide 320 by 4, and by 3, you'll have 26 ells *Flemish*, 2 qrs.

LONG-MEASURE.

Quest. 16. In 927 miles, how many yards, feet, and inches?

	927	Miles
In 1 mile are	1760	Yards
	55620	
	6489	
	927	
	1631520	Yards
	3	Feet 1 yard
	4894560	Feet.
	12	
	58734720	Inches

PROOF.

In 58734720 inches, how many feet, yards, and miles.

Divide back by the same figures, you'll have at last 927 miles.

Note, I shall give you a fuller example in this rule, when we come to *Compound Reduction*, there being *Division* required when you are to bring rods into yards.

LAND-

LAND-MEASURE.

Quest, 17. In 54 acres, how many roods and poles?

54 acres
4 roods 1 acre, *by* 4 or multiply 54 by 160, the rods in an acre, give 8640 rods.

216
40 rods 1 rood, *by* 40

rods 8640

PROOF.

In 8640 rods or poles, how many acres?
Divide by 40, and then by 4, or divide 8640 by 160, and it gives 54 acres.

SQUARE-MEASURE.

Quest. 18. In 217 square yards, 5 feet, how many square feet, inches, and square quarters?

	Square yards Feet	
	217 - 5	
9 square feet 1 yard, *by*	9	
	1958	square feet
144 inches 1 foot	144	
	7832	
	7832	
	1958	
	281952	square inches
16 square qrs, 1 inch	16	
	1691712	
	281952	
	4511232	square quarters.

PROOF.

PROOF.

In 4511232 square quarters, how many square inches, feet, and square yards.

Divide this number back by 16, you have square inches; then by 144, you have square feet, and *lastly*, by 9, you'll have 217 yards, 5 feet.

Note, That 12 times 144, or 1728 solid inches, make a *solid foot*; so that you are to multiply *solid feet* by 1728, to bring them into *solid inches*, and on the contrary, to bring solid inches into *solid feet*, you must *divide* by 1728.

TIME.

Quest. 19. How many days, hours, minutes, and seconds are expired since the birth of our Lord and Saviour, *Jesus Christ*, supposing it 1767 years, 217 days, 17 hours, and 35 minutes, and allowing just 365 days to a year.

Years	*days*	*hours*	*minutes*
1767 -	217 -	17 -	35

```
     365
  ------
    8842
   10603
   5303
  ------
  645172 days
      24
 -------
 2580695
 1290345
 -------
15484145 hours
      60
 ---------
929048735 minutes
       60
 -----------
55742924100 seconds
```

PROOF.

RPOOF.

In 55742924100 ſeconds, how many minutes, hours, days and years?

Divide this by 60, 60, 24, and 365, and you will find 1767 years, 217 days, 17 hours, 35 minutes.

‡ *Note,* According to your tables in time, 365 days 6 hours make a year; therefore, as many years as is given, you muſt multiply them by 6, and add them to the hours.

Queſt. 20. If a lad be juſt 12 years old, how many hours are ſince expired?

12 years	12 years
* 365	‡ 6 odd hous in 1 year
4380 days	72 odd hours in 12 years
24	
17520	
8760	
105120 hours	
add 72 odd hours	
105192	

* *Note,* Though I ſet 365 under 12, yet I multiply 365 by 12, becauſe ir is done in one line.

SEC-

SECTION II.

Of COMPOUND REDUCTION.

Tyro. WHY do you call this *compound reduction?* *Philo.* Because it is compounded of *multiplication* and *division*, and cannot be done without both.

Tyro. What is its particular use?

Philo. It teaches us to reduce any sort of foreign money into pounds *Sterling*, and the contrary; so that it is useful in all business and traffic, as appears by the following table aud examples.

A select ALPHABETICAL TABLE *of foreign coins.*

NAMES	*Value Sterling.*	*What country.*
Abashee	16 pence	*Persia*
Asar	6*s.* 8*d.*	*Ditto*
Asper	3 farthings	*Turkey*
Besse	3 halfpence	*Persia*
Capan	3 pence	*East India*
Capeck	1 penny	*Muscovy*
Cohan	30 shillings	*Japan*
Christiana	16 pence	*Sweden*
Copstake	1 shilling	*Germany*
Crusado	6*s.* 2*d.*	*Ditto*
Ditto	2*s.* 10*d.*	*Portugal*
A Crown	4*s.* 6*d.*	*France*
Ditto	5*s.* 3*d.*	*Florence*
Ditto	7*s.* 6*d.*	*Rome*
Ditto	1*l.* 2*s.*	*Barcelona*
A Dina	1*l.* 10*s.*	*Aleppo*
A Dollar	4*s.* 6*d.*	*Italy*
A Lion Dollar	4 shillings	*Aleppo*
A Cross Dollar	4*s.* 2*d.*	*Holland*
A *Specie* Dollar	5 shillings	*Ditto*
A *Zealand* Dollar	3 shillings	*Ditto*

Old

NAMES.	*Sterling.*	*Country.*
Old Philip's Dollar	5 shillings	*Holland*
Leopold's Dollar	4*s*. 3*d*.	*Ditto*
Rudolphus's Dollar	4*s*. 4*d*.	*Ditto*
Prince of *Orange's* Dollar	4*s*. 4*d*.	*Ditto*
Maxamilian's Dollar	4*s*. 5*d*.	*Ditto*
Ferdinando's Dollar	4*s*. 3*d*.	*Ditto*
Dollar	2*s*. 3*d*.	*Dantzick*
Dollar	2*s*. 3*d*.	*Sweden*
A Rix-Dollar of the empire	4*s*. 3*d*. $\frac{3}{4}$	*Germany*
A Ducat	4*s*. 8*d*.	*Hungary*
Ditto	4*s*. 8*d*.	*Poland*
A *Valentia* Ducat	5*s*. 3*d*.	*Spain*
A *Saragoza* Ducat	5*s*. 6*d*.	*Ditto*
A *Barcelona* Ducat	6 shillings	*Ditto*
Ducat	5 shillings	*Naples*
A Ducatoon	6*s*. 3*d*.	*Holland*
A piece of Eight in common	4*s*. 6*d*.	*Spain*
A *Mexico* piece of Eight	4*s*. 6*d*. and 4*s*. 4*d*. $\frac{1}{2}$	*Ditto*
A *Peru* piece	4*s*. 5*d*.	*Ditto*
A Florin	4*s*. 4*d*.	*Ditto*
Ditto	3*s*. 4*d*.	*Germany*
Ditto	2*s*. 6*d*.	*Sicily*
Ditto	2 shillings	*Holland*
Greven	1 shilling	*Muscovy*
Guilder, or Gilder	3*s*. 8*d*.	*Germany*
Ditto gold	4*s*. 9*s*.	*Ditto*
Ditto of *Noremberg*	7*s*. 1*d*.	*Ditto*
Ditto	5 shillings	*Portugal*
Ditto	18 pence	*Dantzick*
A Harpur	9 pence	*Irish*
A Livre is 20 Sous	18 pence	*France*
A Mark	13*s*. 4*d*. not	*England*
Ditto	current	
Ditto	16 shillings	*Denmark*
	2 shillings	*Germany*
Mark-Lups	3*s*. 9*d*. $\frac{3}{4}$	*Poland*
Messe	15 pence	*East Indie*

Mill-

NAMES.	*Sterling.*	*Country.*
Mill-Ree (is 1000 Rees) -	6*s.* 9*d.* nearly	*Portugal*
A Moidore, marked 4000, or 4 times 6*s.* 9*d.* *viz.* -	27 shillings	*Ditto*
An Obb, or Cobb ½ a Harpur	4*d.* ½ penny	*Irish*
A Pattacoon, or Pettavoon	4*s.* 8*d.*	*Spain*
A Ruble — — —	10 shillings	*Muscovy*
Seraph — — —	5 shillings	*Turky*
Timph — — — —	7 pence	*Poland*
Tical — — —	2*l.* 16*s.* 3*d.*	*China*
Tari — — —	5 pence	*Sicily*
Toman gold — —	3*l.* 6*s.* 8*d.*	*Persia*
Zachin — —	7*s.* 6*d.*	*Venice*
Zelot — —	7*d.* ½ penny	*Turky*

Here follows another useful table.

A TABLE *of the just weight of the customary gold coins used in* England.

	Dwts.	*Grains.*
A Port	9 -	5
½ Port	4 -	14
A Moidore	6 -	22
½ Moidore	3 -	11
Guinea	5 -	9
½ Guinea	2 -	16
A Pistole	4 -	8

Note 1. These all weigh something more than is set down in the table; but as they are fractional parts I have omitted them.

Note 2. If a guinea don't want above 7 grains, a moidore above 9, and a six and thirty above 12 grains of weight, you may safely take them.

Note 3. Scales and weights for this purpose are made by Mr. *John Kirk* in *St. Paul's Church-Yard, London.*

Tyro. I am obliged to you, sir,

Philo. And now, *Tyro*, I will give a variety sufficient for any careful learner to do any question relating to the common

common courſe of buſineſs: but before you begin, I would deſire you to mind the two following obſervations.

Obſervation 1. When one ſum of money, or any weight or meaſure, is to be brought into another ſum of a different name or denomination, then always remember, that before you divide, you make the *Diviſor* and *Dividend* of the ſame name. That is, if the *Dividend* be pounds, pence, yards, ſhillings, &c. your *Diviſor* muſt alſo be of the ſame name.

Obſervation 2. Take notice likewiſe, that your *Remainder* is always of the ſame name as your *Dividend* and *Diviſor*, be it what it will. Theſe obſervations kept in memory will be of great ſervice to you in the *Rule of Three direct*, and in all common rules of arithmetic.

1. Queſtions *in* Compound Reduction.

In 324 moidores how many pounds?

	324	moidores
1 moidore	27	ſhillings
	2268	
	648	
Shillings 2|0)	874|8	ſhillings
	£. 437 - 8	ſhillings

2. A Merchant at *London* delivers to his correſpondent as much broad-cloth as comes to £. 547 - 14 - 7; and he is to receive the ſame in croſs dollars, at 4*s.* 2*d.* each. How many muſt he receive?

Rule. Bring the money into pence, and divide by the pence in 1 dollar.

s. *d.*	
4 - 2	£. 547 - 14 - 7
12	20
—	10954 ſhillings
50	12

Pence in a dollar 5|0)13145|5 pence

26291 croſs dollars. *Anſ.*

3. How many cruſadoes, at 6*s.* 2*d.* each, muſt be paid for 1000 guilders of *Noremberg*, at 7*s.* 1*d.* each?

	s. *d.*		
Cruſado	6 - 2	1000 guilders	*s.* *d.*
	12	85	7 - 1 guilder
	—	74)85000(1148	12
	74 pence	74	—
		110	85 pence
		74	
		360	
		296	
		640	
		592	
		48 pence	

Anſ. 1148 cruſadoes and 4 ſhillings

Quest. 4. How many rix dollars, at 4*s.* 3*d.* ¼ each, may I receive for £ 1750 - 15 - 6½ sterling?

Rule. Bring the sterling money into farthings, and divide by the farthings in 1 dollar.

```
  s.  d.             £ 1750 - 15 - 6½
   4 - 3¼                  20
  12                 -----------
 ----                  35015 shillings
  51                       12
   4                 -----------
 ----                 420186 pence
 205                        4
                     -----------
Farthings in 1 dollar 205)1680746(8198 rix doll. Ans.
                          1640      and 156 farth.
                          -----       or 3s. 3d.
                           407
                           205
                          -----
                           2024
                           1845
                          -----
                            1796
                            1640
                          -----
                      by 4)156 farthings over *
                          -----
                     by 12)39 pence
                          -----
                           3 - 3 pence
```

Tyro. * The *Remainder* I see is farthings as well as the *Dividend*; and I suppose you divide it by 4, and by 12, to bring it into pence and shillings: Do you not?

Philo. You say right; and all other questions are done after the same manner.

Quest. 5. In 276 crusadoes, at 6*s.* 2*d.* each, how many *Hungarian* ducats, at 4*s.* 8*d.* each?

```
  s.  d.               276 crusadoes
  4 - 8                 74 pence in a crusado.
 12                   ----                 s.  d.
 --                   1104                 6 - 2
 56                   1932                12
                      ----                --
Pence in a ducat   56)20424(364 ducats Ans. 74 d.
                      168
                      ----
                       362
                       336
                       ---- Ans. 364 ducats, and
                        264   40 pence over, or
                        224   3s. 4d.
                        ----
                        40 pence over
```

Quest. 6. In £. 347 - 15 - 7, how many *French* crowns, at 4*s.* 6*d.* $\frac{1}{8}$ each?

```
                     £ 347 - 15 - 7
                            20
                      -----------
                       6955 shillings
        d.                  12
A crown 54 1/8        -----------
         8             83467 pence
      ----                   8 eighths in 1 penny
433 eighths           -----------
                 433)667736(1542  Ans.   1542 French
                     433              crowns and 6d. 2/8 1/4
                     -----------
                     2347                Here I multiply the
                     2165             pence by 8, and the
                     -----------      Dividend and the Divi-
                      1823            sor an eighth of a pen-
                      1732            ny, the Remainder also
                     -----------      is 8ths of a penny;
                        916           therefore, I divide it by
                        866           8, and it is 10 pence,
                     -----------      and 3 eighths over.
                       8(50
                         6 2/8  1 1/4
```

Note 1. As you do not underſtand fractions as yet, it may not be amiſs to give you a word or two by way of information. You ſee a *French* crown in the above example is valued at 54*d.* 1 eighth, that is, 54 pence, and 1 eighth of a penny, which 1 eighth is called a fraction; the 1 which ſtands a-top, is called the numerator, and the 8 is the denominator: ſo alſo $\frac{3}{7}$ is read 3 ſevenths, $\frac{5}{11}$ is 5 elevenths, and $\frac{3}{5}$ is 3 fifths. Here 2. 5, and 3 are numerators, and 7, 11, and 5, are the denominators.

Note 2. The rule to work by fractions in all such cases as this, is thus: Multiply the whole number by the *Denominator*, that is, the lower figure of the fraction, and take in the top figure, or *Numerator*.

Quest 7. A Merchant sends to his correspondent as much corn as comes to £ 1575, to receive the same in ducatoons, at 6*s*. 3*d*. $\frac{3}{5}$ each; how many ought he to receive?

```
s.  d.                              £ 1575
6 - 3 3/5                               20
12                                 -------
----                                 31500 shillings
75                                      12
by 5 and take in the top 3          -------
----                                378000 pence
378 fifths                     by        5 brings d. into 5ths
                                    -------
            Divisor 378)1890000(5000 ducatoons
                        1890              Ans.
                        -------
                         000
```

Quest. 8. In £ 1000 how many 7 pence halfpennies?

```
                £ 1000
                    20
                ------
                 20000 shillings
                    12
                ------        Or thus :
                240000 pence   960 farth.
                     4        1000
                ------        ------
Farthings in 7 1/2 are 3|0(96000|0   3|0)96000(0
                ------        ------
                 32000 Ans.    3200 Ans.
```

Quest.

Quest. 9. In 100 *Portugal* pieces, at 36 ſhillings each, how many pounds, half crowns, and crowns?

Piece 36 ſhillings
100

2[0)360[0 ſhillings

180 pounds
8 half crowns 1 pound

2)1440 half crowns

720 crowns

Here you ſee I multiply, or divide always by as many of the *leſs* as make one of the *greater*, according as the nature of the queſtion requires.

Quest. 10. In 432 moidores, how many shillings, crowns, groats, and farthings?

```
                          432 moidores
                           27 shillings 1 moidore
                         ----
                         3024
                         864
                         ----
Shillings in 1 crown 5)11664 shillings
                         ----
       Crowns            2332 4s. over, or 12 groats.
Groats in 1 crown          15 and take in 12 groats.
                         ----
                        11662
                        2333
                        -----
                        34992 groats.
Farthings in 1 groat       16
                        -----
                       209952
                       34992
                       ------
                       559872 farthings
```

Tyra. I see now, Sir, the nature of the rule plainly; but pray how do you manage when there is a sum of money to be paid in pieces of different value, and that there shall be as many in number of one sort as of another; that is, an equal number of every sort?

Philo. This is very useful in many respects, and is done by this general rule, *viz.* Add all the pieces together; then bring the sum of money to be paid, and the sum of all the pieces into one name or denomination; and *lastly*, divide the sum to be paid by the total of all the pieces, and the quotient is the number of each, one with another.

Quest.

Quest. 11. How many ſhillings, 6 pences, 4 pences, 3 pences, 2 pences, pence, half-pence, and farthings, of each a like number, will diſcharge a debt of £335 - 8 - 4? *Anſ.* 2800 of each.

I ſet down all the pieces one under another, as follows:

A ſhilling is	12	
	6	
	4	
	3	£335 - 8 - 4
	2	20
	1	
	$\frac{1}{2}$	6708 ſhillings.
	$\frac{1}{4}$	12
Pence	$28\frac{3}{4}$	80500 pence
	4	4
Farthings	115	115)322000(2800 pieces of each (ſort
		230
		920
		920
		00

Here I bring £335 - 8 - 4 into farthings, and divide by 115, the farthings in all the pieces, and the anſwer is 2800 of each.

OBSERVATION I.

1. When any pieces of money are to be brought into pounds *ſterling*, you ſhould conſider what relation thoſe pieces bear to a pound *ſterling*; that is, what part of a pound it conſiſts of; for by this the work will be rendered much ſhorter, and you will not have occaſion to

reduce the work into pence: but of this I ſhall ſhew you more fully in the *Rule of Three Direct*.

Queſt. 12. In £ 270 how many nobles, at 6*s*. 8*d*. each?

270
  3 nobles, or 3 times 6*s*. 8*d*. is 1 pound

810 *Anſ*.

To bring nobles into pounds, divide by 3.

Queſt 13. In £ 270 how many marks, at 13*s*. 4*d*.

Here, as 13*s*. 4*d*. is 2 thirds of a pound, I multiply the pounds by 3, and divide by 2, as follows.

270
  3

2)810

405 marks. *Anſ*.

Tyro. This is much ſhorter than to bring the *pounds* and the *marks* into pence, and have to divide beſides.

Philo. To be ſure it is, and when you want to bring *marks* into *pounds*, then multiply by 2, and divide by 3; for you muſt have leſs *pounds* than *marks*, becauſe a *pound* is larger than a *mark*. And thus, *Tyro*, by your care and obſervation, you may work many ſums ſhorter than the common form of rules in general laid down; for it is impoſſible (without a large volume indeed, and even not then) to lay down rules and examples to make a careleſs learner underſtand: but a diligent learner, that is deſirous to know, and make himſelf perfect, will not only obſerve the rules given him,

him, but will also try and contrive methods of his own, and is not satisfied with knowing things by halves.

Quest. 14. Suppose a bill of exchange was accepted at *London*, for £ 580, for value received at *Amsterdam*, how many *Flemish* pounds ought to be remitted at *Amsterdam?*

```
        33 - 4 one F. pound
        12                £ 580
       ----                  20
Divisor 400 pence        ------
                          11600
                             12
                         ------
                   4|00)1392|00
                         ------
                Ans. 348 Flemish pounds.
```

Do you understand it?

Tyro. I do; for let the question be what it will, I have nothing to do but make them both of a name, and multiply, or divide by as many of the less, as make one of the greater; then the quotient (if a division sum) will be the answer, and the remainder the same as the dividend.

Philo. Very well; then I will set you one sum more for trial.

Quest. 15. A General of an army, (consisting of 5000 men) after a very sharp engagement lost 2380 men; but coming off victorious, he, for their valiant behaviour, gave 1000 guineas to be equally divided among them, and the remainder (if any) to be given to a little errand-boy: How much did each man receive?

Tyro. I proceed thus: I take 2380 men that were lost, out of 5000, and there remains 2620 that escaped. Then I reduce 1000 guineas into farthings, and find it

it 1008000, and dividing this by the number of men that escaped, *viz.* 2620, I find the quotient 384 farthings, that is just 8 shillings each man, and 192 farthings remaining over, which is 4 shillings for the little errand boy, as appears by the following work.

```
                    5000 soldiers
                    2380 killed
                    ----
          Divisor   2620 escaped

        1000 guineas
          21
       -----
       21000 shillings
          12
      ------
      252000 pence
           4
Men   ------
262[0)100800[0(384 farthings each
      786
      ----           4)384 farthings
      2220             ---
      2096         12) 96 pence
      ----             --
       1240             8 shillings each.  Ans.
       1048
      -----
      4)192 farthings remain
      -----
      12)48 pence
      -----
           4 shillings for the little boy.
```

Philo. Very well done indeed.

LONG-

LONG-MEASURE.

Quest. 16. How many furlongs, rods, yards, feet, inches, and barley-corns will reach round the world, supposing it (according to the best calculation) to be 25020 miles.

25020 miles *
8

200160 furlongs
40

8006400 rods
11 half yards in 1 rod †
---
by 2)88070400 half yards

44035200 yards
3

132105600 feet
12

1585267200 inches
3

4755801600 barley-corns

* According to this calculation of 25020 miles round the globe, 69 ½ miles make 1 degree. See the *note* in *page* 49, *long-measure*.

† *Note*, That 5½ yards make 1 rod; but because I cannot well multiply by 5½, I therefore multiply by 11, and they are ½ yards; and then divide by 2 to bring them into yards.

Another

Another way.

As 1760 yards make a mile, if you multiply 25020 by 1760, you will have the same answer in yards; and if you multiply the yards by 36, you will have the inches, &c.

SECTION III.

QUESTIONS *to exercise* REDUCTION.

1. IN £.56 how many *crowns* and *six-pences?* *Ans.* 224 crowns, and 2240 six-pences.

2. In 8568 pence, how many shillings, and guineas? *Ans.* 714 shillings, and 34 guineas.

3. In £.12 - 2 shillings, how many groats, three-pences, and six-pences? *Ans.* 726 groats, 968 three-pences, and 484 six-pences.

4. In 750 pounds, how many moidores and guineas? *Ans.* 555 moidores, and 15 shillings, and 714 guineas, and 6 shillings over.

5. In 500 *ports*, at 36 shillings each, how many pounds, half crowns, crowns, and groats? *Ans.* £.900, 7200 half crowns, 3600 crowns, and 54,000 groats.

6. In £.500 how many pieces, at 7*d.* $\frac{1}{2}$, and pieces at 7*s.* 6*d.* each? *Ans.* 16,000 pieces, at 7*d.* $\frac{1}{2}$. And 1333 pieces, at 7*s.* 6*d.* each, and $\frac{1}{3}$ of a piece, or 2*s.* 6*d.*

7. A merchant at *Holland* draws a bill upon his correspondent in *London*, for 2500 ducatoons, at 6*s.* 2*d.* $\frac{2}{5}$ each, how many pounds *Sterling* ought he to receive at *London*. *Ans.* £.777 - 1 - 8.

Rule. Bring a *ducatoon* into 5ths of a penny, (as in *page* 138) and multiply the number of *ducatoons* by them; then bring a pound *sterling* also into *fifths* (*viz.*

1200

1200 fifths) and dividing by 1200, you will have the £. 777 $\frac{1}{12}$; that is, £. 777 one twelfth of a pound, or 1*s*. 8*d*.

8. A rich *Nobleman* has 5 villages, in every village 3 streets, in every street a dozen houses, in every house 5 rooms, in every room 2 bureaus, in every bureau 12 drawers, in every drawer 4 bags, every bag valued at 150 guineas; which he is going to *exchange* for £. 3 - 12 shilling pieces, how many must he receive in all?

Rule. Multiply all the numbers by one another as they follow in order, and your last product will be guineas. Bring these guineas into shillings, and divide by 72, the shillings in a £. 3 - 12 piece, you'll have the answer, which is 3780000 pieces, at £. 3 - 12 shillings each.

And thus, *Tyro*, may you by dilligence and observation do any sort of sum.

Tyro. Sir, I see it plainly, and will try to do them all, and prove their answers the first opportunity.

Philo. Then I will now proceed to the most beautiful and most useful rule in all common *arithmetic*, *viz*. the *Rule of three direct*, commonly called the *Golden rule of three*.

DIA-

DIALOGUE VII.

SECTION I.

The single Rule of three direct, *commonly called the* GOLDEN RULE.

Tyro. WHAT do you mean by the *Rule of three direct?*

Philo. The *Rule of three* is the *rule of proportion*; because it shews what *relation* or *proportion* one number bears to another.

Tyro. What is given, and what is required in the *Rule of three?*

Philo. *Three* numbers are given to find a *fourth*, which is the answer.

Tyro. How is this *fourth* number found?

Philo. By this one *general rule.* Multiply the second number by the third, and divide by the first.

Tyro. How does the proportion of these 4 numbers stand, or what relation do they bear to each other?

Philo. As the *first* is to the *second* number, so is the *third* to the *fourth* number. For if you multiply the *fourth* number, or *answer* by the *first*, the product will be the same as the *third*, multiplied by the *second:* but more of this by and by.

Tyro. How am I to work the *Rule of three direct?*

Philo. By the following rule, which you ought so far to get by heart, as to understand it without book.

A Ge-

A General RULE.

1. When any 3 numbers are given, place them one after the other, in order, as they stand, and this is called stating the question.

This being done, multiply the *second* by the *third*, and divide by the *first*, and you have the *forth* number, or answer.

Let the three numbers be 4, 8, and 12.

As 4 : 8 :: 12 to a 4th number, *viz*. 24.

$$\begin{array}{r} 8 \\ \hline 4)96 \\ \hline 24 \text{ } \textit{ans.} \end{array}$$

Thus 4 : 8 :: 12 : 24, thus read, as 4 *to* 8, *so is* 12 *to* 24.

Here you see the *fourth* number, or answer, is 24.

Now, *Tyro*, it is worth your while to observe what *relation* or *proportion* these bear to each other.

OBSERVATION 1.

As the *first* is to the *second*, so is the *third* to the *fourth*. That is, as 4 is to 8, so is 12 to 24: for 4 is the half of 8, and 12 is the half of 24.

OBSERVATION 2.

As the *first* is to the *third* so is the *second* to the *fourth*. That is, as 4 to 12, so is 8 to 24: for 12, the *third* number, is 3 times more than 4, the *first* number; and 24, the *fourth* number, is 3 times more than 8, the *second* number. Or, in other Words, thus: the *third* divided by the *first*, is equal to the *fourth* divided by the *second*. And thus you will find the *proportion* holds good,

good, take the numbers which way you will, as by the following *observations*.

OBSERVATION 3.

The *first* number is equal to the product of the *second* and *third*, divided by the *fourth*; that is 96 divided by 24 is equal to 4.

OBSERVATION 4.

The *second* is equal to the product of the *first* and *fourth*, divided by the *third*; that is 96, divided by 12, is equal to 8.

OBSERVATION 5.

The *third* is equal to the product of the *first* and *fourth*, divided by the *second*; that is, 96 divided by 8 is equal to 12.

OBSERVATION 6.

The *fourth* is equal to the product of the *second* and *third*, divided by the *first*; that is, 96 divided by 4 is equal to 24 *.

Tyro. This is very pretty indeed! and will this *proportion* hold so in any other 3 numbers,

Philo. No doubt of that; if you will try any 3, you will find they will answer to the foregoing *observations*.

This is only to shew you, *Tyro*, the nature of proportion, relating to numbers, without any particular name; but as for working the *Rule of three direct*, when the numbers are called by the names of *pounds*, *yards*, *ells*, *ounces*, *pence*, or the like; that indeed is the effect of practice, and may very soon be learned by the following instructions, or directions.

* See more of this in my *Young algebraist's companion*, dialogue 7, section 2.

SEC-

SECTION II.

Containing some full and necessary directions for the better performing this rule.

Tyro. YOU have already told me, that there are always *three numbers* given to find a *fourth*; but in what order am I to place them, when they are of a different name, or denomination. That is, *how am I to state the question?*

Philo. Do you but mind the following instructions, and read your questions over slow, and considerately, and you will soon know how to *state* it, or place the numbers in their order; for I shall first of all give you directions to work common easy sums, and then shew you how to proceed when the *first* and *third* numbers are not alike, and when there is required two or three statings.

SECTION III.

Directions *to state and work easy Questions in the* Rule of three direct.

First, ALWAYS remember that when 3 numbers are given, *two* of these numbers always are of one and the same name and denomination, *viz*, either money, weight, or measure, which *two numbers* out of the *three*, must be your *first* and your *third* number; and the other in course must be placed in the middle, which *middle number* will always be of the same name as the answer. Thus if the *second* or *middle number* be *money*, *weight*, or *measure*; so also will the *fourth number* or *answer* be.

2. If

2. If at any time the *first number* or the *third*, be of a higher or grerter denomination than the other, then make them both of one denomination by the rules of *reduction*. Thus, if the *first number* be yards, and the *third number* ells, reduce them both into quarters, by multiplying the yards by 4, and the ells by 5; so also if the *first number* be *lbs.* and the *third Cwts.* and *qrs.* you must reduce the *Cwts.* and *qrs.* into the *lbs.* that they may not only be of one name, (weight, measure, &c.) but likewise the very same name, *viz. pounds*, *pence*, *yards*, *quarters*, &c.

3. If your *second number* be a compound quantity, such as *pounds*, *shillings*, and *pence*, or *Cwts. qrs.* and *lbs.* you must reduce it into pence, or into *lbs.* by the rule of *reduction*; and remember, that after you have multiplied the *second* by the *third*, and divided by your *first*, the quotient will be of the same name as you reduced your *second* number into, be it what it will.

4. In all questions in the *Rule of three*, where there is only 3 numbers given, you have already been told, that *two* of those *three* will always be of one name, and must be your *first* and *third* numbers. Now, it is very easy to know which of those *two* must be your *third* number: for whenever a question is asked, or whatsoever number follows the question, *that* is your third number. Thus, what cost 24 yards? or how much does 96*lb.* cost? Here 24, or 96, will be your *third* number; and so for any other question, as you will see by the two first examples, as follows.

Quest.

Quest. 1. If 3 yards cost 7 shillings, what cost 24 yards?

Yds. shill. yds.
If 3——7——24
7 multiply
Divide by 3)168
by 2|0) 5|6 shillings
2*l.* 16*s.* *Ans.*

Proof, or proportion.
As 3 : 7 : 24 : 56. For 3 times 56 is 168, and 7 times 24 is 168. And thus for any other question.

Quest. 2. If 8 ells cost 11 shillings, what cost 96 ells?

Ells shill. ells
If 8——11——96
11
8)1056
2|0) 13|2
£. 6 - 12 *Ans.*

☞ *Note*, Here you see that the question is asked, what cost 24 yards, or what cost 96 ells; therefore, you may be sure these are to be your third number. Then your *first* is plainly known by being of the same name; and your *middle*, or *second* number, must be known of course, which will always be same name as the answer. See *observation* 1.

Rule. Having stated the question, I multiply the *second* by my *third* number, or the *third* by the *second*, and divide by the *first*, and the answer is 56 shillings, which is the same name as the *second* number; then I divide

divide those shillings by 20, and have the answer. The second question is done the same.

Tyro. I see the nature of it plainly; but there is a way to prove sums in the *Rule of three*, by stating them backward, is there not?

Philo. Yes; and a very good method it is to confirm you in the truth of what you have done; for the question will then be turned quite the reverse way.

First proof, or variation.

If 24 yards cost 56 shillings, what cost 3 yards?

```
   Yards  shill.  yards
If 24 ——— 56 ——— 3
            3
          ———
      24)168(7 shillings.  Ans.
         168
         ———
           0
```

Here you see the *fourth* number in the original question, is the *first* number in this question; the answer of the original is the *second* number; and the *second* number in the original is the *answer* here.

Second proof, or variation.

If 7 ſhillings buy 3 yards, how many can I have for 56 ſhillings?

Here I find 2 numbers out of the 3 are money, and the number following the queſtion (*how many*) is 56 ſhillings; therefore, 56 ſhillings is my third number; which is thus ſtated.

	Shill.	*Yards*	*Shill.*
If	7 —	3 —	56
			3
			7)168
			24 yards. *Anſ.*

So that you ſee, by minding which *two* of the *three* are of the ſame name, it is no difficult matter to ſtate any common queſtion, and others more difficult will naturally follow by uſe and practice.

Tyro. I underſtand it very well indeed, and heartily thank you.

Philo. Then I ſhall proceed to give you more uſeful examples, and when any thing occurs that you don't rightly apprehend, aſk me, or read the inſtructions over again.

Quest. 3. If 3 *lb.* of tobacco, (or any other thing) cost 2*s.* 9*d.* what cost 255*lb.*?

```
   lb.   s.   d.        lb.
If 3 ——— 2 ——— 9 ——————— 255
         12               33  second number
         —              ————
         33               765
                         765
                        ————
                        3)8415
                        ————
                       12)2805 Pence
                        ————
                       2|0)23|3-9
                        ————
                     £. 11 - 13 - 9 Ans.
```

PROOF.

```
    Pence     lb.     Pence
If 2805 ——— 255 ——— 33
                33
               ————
               765
              765
             ————
        2805)8415(3lb. An
             8415
             ————
               0
```

☞ The *second number* being *shillings* and *pence*, I bring them into pence by multiplying by 12, and taking in the odd 9 pence; and after I have multiplied the *third number* 255 by 33, and divided by my *first*, which is 3, the quotient is of the same name still, *viz.* pence, which

which I divide by 12, to bring into ſhillings, and by 20 to bring them into pounds, as above; and thus for any common ſum. See *obſervation* 2.

Queſt. 4. If an ounce of tobacco coſt 5 farthings, what coſt 2 *C*. 3 *qrs*. 21 *lb*.?

```
    oz.  farth.  C.    qrs.   lbs.
If *1 ——— 5 ——— 2 ——— 3 ——— 21
                4
             ———
               11 qrs.
               28
             ———
               89
               24
             ———
              329 lbs.
               16
             ———
             1974
             329
             ———
             5264 oz.
                5 second number
             ———
          4)26320 farthings
             ———
          12)6580 pence
             ———
          2|0) 54|8-4d
             ———
          £.27 - 8 - 4 Anſ.
```

* *Note*, Becauſe the *firſt* number is ounces, I bring the *third* number alſo into ounces, to anſwer it; but after I have multiplied by my *ſecond* number 5, they are all farthings; and as my *firſt* number is only an unit (or 1) I do not divide by *that*; for 1 neither multiplies nor divides: I therefore only divide the farthings by 4, 12, and 20, and the queſtion is done. See the *proof* the next queſtion.

Quest. 5. If 2 C. 3 *qrs.* 21 *lb.* (or 5264 ounces) cost £. 27 - 8 - 4, what cost an ounce?

```
     oz.        £.  s.  d.     oz.
If 5264——27 - 8 - 4——1
                 20
             ———
                548
                 12
             ———
               6580
                  4
             ———
   5264)26320(5 farth. Ans.
          26320
             ———
                  0
```

☞ Take notice that whenever your *first* number is large, and the *third* small (as in this example) that you always reduce the *middle*, or *second* number, to its lowest denomination.

Quest.

Quest. 6. Bought a silver cup and tankard, which weighed 3 *lb.* 11 *oz.* *troy*, and cost me £. 8 - 18 - 2½; what did they lie me in *per* ounce?

Here, as the question is, *What did they cost* per *ounce?* You must know your answer is to be in money; so also must your *second* number be, and the other two numbers will naturally follow, thus:

```
    lb. oz.               £.  s.   d.           oz.
If  3 - 11 ————————————— 8 - 18 - 2½ ————————— 1
   12 oz. 1 lb. troy     20
   ——                   ———
   47 oz.               178 shillings
                         12
                        ————
                        2138 pence
                           4
                        ————
               47 47)8554(182 farthings: Ans.
                     47
                    ———          4)182 farthings
                     385         ————
                     376        12) 45-2
                    ————        ————
                      94                3s.9d.½ per oz. Ans.
                      94
                    ————
                       0
```

Quest. 7. A gentleman has an estate which brings him in clear £. 2000 a year, and the charges he is at, one day with another, for house-keeping, and other expences, is £. 5 - 15*s.* I demand what he loses or saves every year?

Rule. See what £. 5 - 15 *per* day amounts to in a year, and take it out of his yearly income £2000 is what he saves; but if his expences be more, take the £. 2000 from that, and the remainder is what he loses.

	Day	*£.*	*s.*	*days*	
If	1 ——	5 -	15 ——	365	
		20		115	
		115		1825	
				4015	
				2|0)4197|5	shillings

Yearly expences £. 2048 - 15

Yearly expences	——	£. 2048 - 15
Estate	———	2000 -
He loses yearly	———	£. 48 - 15 *Ans.*

Quest.

Quest. 8. A farmer agreed with his servant to thresh all the corn he had, and the servant was to receive a guinea for every 7 quarters: now, he threshed in all 15 loads, 1 quarter, and has received of his master, at different times, by cash and goods, 9 guineas; I demand how the reckoning stands between them.

Rule. First, see what 15 loads 1 quarter comes to, at a guinea, or 21 shillings, per quarter, and then subtract 9 guineas out of it,

```
   qrs.    s.    loads qr.
If 7 —— 21 —— 14 — 1
                 5 qrs. 1 load
              ——————
                76 qrs.
                21
              ——————
                76
               152
              ——————
             7)1596
              ——————
          2|0) 22|8 shillings
              ——————
           £. 11 - 8 Ans.
```

Servant's wages for threshing	——	£. 11 - 8
He received 9 guineas, *viz.*	——	9 - 9
Due to the servant	——	£. 1 - 19 *Ans.*

Quest. 9. A corn-factor sends to his correspondent in *Spain*, 10,000 quarters of wheat, and agreed at £. 8 - 7 - 6 per load, and he has received by several remittances £. 10,000; what is still due to him?

Load	£. *s.* *d.*	*qrs.*
If 1 ——	8 - 7 - 6 ——	10,000
5	20	
5 *qrs.*	167	
	12	
	2010 pence	
	10000	
First number 5)	20100000	
12)	4020000 pence	
2|0)	335000|0 shillings	
	£. 16750 *Ans.*	

Wheat	£16,750
Received	10,000
Due to him	£6,750

Quest.

Quest. 10. Bought 4 bags of hops, which weighed as under

		C.	qrs.	lb.
No	1 —	2 -	3 -	11
	2 —	1 -	2 -	27
	3 —	2 -	1 -	17
	4 —	1 -	3 -	15
		8 -	3 -	14

And gave after the rate of 5 guineas *per cwt.* what do they come to?

C.	£. s.	C. qr. lb.
If 1 ———	5 - 5	8 - 3 - 14 * *
112	20	4
112 *lbs.* 1 *dwt.*	105	35 *qrs.*

```
      35 qrs.
      28
     ---
     284
     71
     ---
     994 lbs.
     105
     ---
     4970
    9940
First number 112)104370(931 shillings divided
     1008          by 20, is £.46 - 11.
     ----          So that the answer
      357          is £.46 - 11 - 10½
      336
      ---
      210
      112
      ---
       98 Remainder
       12
      ---
112)1176(10 Pence
     112
     ---
      56 Remainder
       4
     ---
112)224(2 Farthings
     224
       0
```

 * * Note,

** *Note*, I reduce the first and third into *lbs*. and the second into shillings, and the quotient is shillings; and the remainder 98 I multiply by 12, and divide by the same divisor, (*viz*. 112) and it gives 10 pence in the quotient; and the next remainder I multiply by 4, and it gives 2 farthings.

** ☞ See a shorter way than either this, or page 121, to bring *cwts*. *qrs*. and *lbs*. into *lbs*. *viz*. in example 1, in tare and tret.

Quest. 10. There is a ship worth £. 2000, of which I have the $\frac{7}{32}$ parts, I demand what my share is worth?

First, according to the common order of the rule of three, I say,

```
 Parts                         Parts
If 32 be —— £. 2000 what is 7
                  7
               ————
          32)14000(437 pounds
             128
             ————
              120
               96
             ————
              240 Ans. £. 437—10 s.
              224
             ————
               16 remainder
               20 shillings in a £.
             ————
          32)320(10 shillings
              320
             ————
               0
```

Note,

Note, I multiply the remainder 16 by 20, and divide by the ſame *diviſor* 32, and the quotient is ſhillings. Had any thing remained I ſhould now have multiplied it by 12, and divided by 32, and the quotient would have been pence, &c.

Note, It often happens, that dividing by the parts is both ſhorter and eaſier than dividing by the whole: nothing but practice and obſervation can make a man maſter of this; for ten thouſand rules without theſe are of very little account.

OBSERVATION.

Here, according to the nature of queſtions of *multiplication*, or *diviſion* of money, I perform the above queſtion with eaſe.

Parts		*Parts*
If 32 ——————	£ 2000 ——	7
	7	
8 times 4 is 32, *by*	8)14000	
	4)1750	
	Anſ. £437 - 10	

Here I divide by 8, and by 4; when I divide by 4, I find 2 remain, which I call 2 pounds; then I ſay the 4th part of £ 2 is 10 ſhillings. And now you may ſee the beauty of *multiplication* and *diviſion of money*, if you do but obſerve it well.

Tyro. I humbly thank you, Sir, and ſee there is nothing like knowing the former rules perfectly.

Philo. As I ſaid before, many queſtions may be performed by *multiplication*, and *diviſion of money*,

if they be rightly applied, and much ſhorter and eaſier than the common order of the *Rule of Three*. I ſhall give you one inſtance, and no more, which, if well obſerved, will be ſufficient; for according to the nature of the queſtion, a great many figures may be ſaved as well as trouble.

Queſt. 11. Bought a box of ſuperfine tea, which weighed 2 *qrs*. 7 *lb*. and gave £ 73 - 12 - 7 ½ for it; what is 5 *lb*. of it worth?

Firſt, and common way.

```
    qrs.  lb.          £.   s.   d.       lb.
 If  2 —— 7 ———————— 73 - 12 - 7½ ———— 5
    28                 20
    ——               ————
    63               1472 ſhillings
                       12
                     ————
                    17671 pence
                        4
                    —————
                    70686 qrs.
                        5 third number
                   ——————
               63)353430(5610 farthings
                  315
                  ———               Farthings
                   384               4)5610
                   378               ————
                  ————              12)1402-½
                     63              ————
                     63            2|0)11|6-10
                  ————              ————
                      0           Anſ. 5-16-10½
```

Short-

Short way by multiplication *and* division of money.

	lb.	*£.* *s.* *d.*		*lb.*
If	63 —	73 - 12 - $7\frac{1}{2}$	—	5
		5		
63 is 9 times	7)	368 - 3 - $1\frac{1}{2}$		
	9)	52 - 11 - $10\frac{1}{2}$		
		5 - 16 - $10\frac{1}{2}$	*Anf.*	

Here you see that the second way has but about half the figures.

Tyro. I see it very plainly; and I think I am able to do any common sum of but one *stating*.

Philo. Then we will proceed to *section* 4.

SECTION IV.

Containing some necessary directions in more difficult questions, and such as require more than one stating.

Tyro. HOW am I to manage when the *first* and *third* numbers are not of one name, or denomination.

Philo. Bring them into one denomination (according to *direction* 2, *section* 1.) and proceed by multiplying your *second* by the *third*, and divide by your *first*, as before.

Quest.

Quest. 12. If 3 yards cost 8*s.* 3*d.* what cost 96 ells *English.*

```
Yards            s.   d.  Ells Engl.
If 3 ———————— 8 - 3 ———96
   4            12          5
   —            —        ————
12 qrs.         99         480 qrs.
                            99
                          ————
                          4320
                         4320
                         —————
First number        12)47520
                         —————
                    12) 3960 pence
                         —————
                    2|0) 33|0
                         —————
                      £ 16 - 10.  Ans.
```

Rule. Here, because the *first* number is yards, and the *second* ells *English*, I bring them both into quarters, by multiplying the yards by 4, and the ells by 5, and then proceed as before.

First

First proof, or variation.

Quest. 13. If 96 ells *English* cost £ 16 - 10*s.* what cost 3 yards?

Ells	£. s.	*Yards*
If 96	16 - 10	3
5	20	4
480 qrs.	330 shillings	12 qrs.
	12	
	3960 pence	
	12 third number	

First number 480)47520(99 pence. *Ans.* or 8*s.* 3*d.*
4320
———
4320
4320
———
0

☞ Here you see the *fourth* number or answer, is the same as the *second* number, in the last question. See the next question.

Quest.

Second Variation, or Proof.

Quest. 14. If I buy 3 yards of holland for 8*s.* 3*d.* how many ells *English* may I have for £. 16—10.

☞ Here, because the answer is to be in measure, therefore measure must be my second number, thus:

```
   s.  d.  Yards.        £.  s.
If 8 - 3.—— 3 ——————16 - 10
   12                 20
   ——               ————
   99 pence          330 shillings
                      12
                     ————
                    3960 pence
                       3 second number
                   ——————
First number 99)11880(120 yards
                 99
               ————
                 198
                 198           Yards.
               ————             120
                   0              4 qrs. 1 yard
                               ————
             Qrs. 1 ell 5)480 qrs.
                           ————
                            96 ells.  Ans.
```

☞ Here you see I do not bring the second number, yards, into quarters, as I did before in the other two questions; for it is better to work with it as it is; because it saves a great many figures, and the answer will be 120 yards, which I reduce into ells very easily, by multiplying by 4, and dividing by 5; and if you consider all the three different ways of working this question, it will be of great service to you.

Quest.

Quest. 15 A Draper bought 12 pieces of holland, each piece containing 23 ells *English*, and gave after the rate of 3*s*. 4*d*. per ell *Flemish*; what did they come to?

First 23 ells 1 piece
12 pieces
———
267 ells in all

Ell Fl.	*s. d.*	*Ells Eng.*
If 1 ———	3 - 4 ———	276
3 qrs. 1 ell *Fl.*	12	5
3 qrs.	40 pence	1380 qrs.
		40
		3)55200
		12)18400 pence
		2|0) 153|3 - 4
		£. 76 - 13 - 4 *Ans.*

Tyro. I am obliged to you; and now, if you please, I will try at one.

Philo. with all my heart; I like to see you in this mind.

Quest.

Queſt. 16. A Woollen-Draper bought 4 packs of cloth, each pack containing 3 parcels, each parcel 6 pieces, and each piece 40 yards, and gave 3 *l.* 10 *s.* for every 6 yards one with another; what do they come to?

Tyro. Firſt, I ſee how many yards there are in all, by multiplying them together as they ſtand, thus:

4 packs	*Yards* £. *s.*	*Yards.*
3	If 6—3—10	—2880
——	——20	70
12 parcels	—	——
6	70	6)201600 ſhillings
——		——
72 pieces		2|0) 3360|0
40		——
——		1680 *Anſ.*
2880 yards in all		

Philo. Very right; and you may prove this ſum ſeveral ways at leiſure, which will be of ſervice. Firſt, £. 3 - 10 *s.* every 6 yards is 11 *s.* 8 *d.* for 1 yard. Therefore ſay, if 1 yard coſt 11 *s.* 8 *d.* what coſt 2880, and it will be the ſame anſwer. And again, you know 2880 yards, at £ 1 per yard, is £ 2880; therefore ſay, If 20 ſhillings be 11 *s.* 8 *d.* what will £ 2880 be; and you will have 1680 for anſwer, as before.

Queſt.

Quest. 17. How many dozen of candles, at 5*s.* 3*d.* per dozen, may I have in *exchange* for 3 pieces of *Irish*, each long 20 ells, at 3*s.* 9*d.* per yard?

Here is two ſtatings in this queſtion; firſt, ſee what the *Iriſh* comes to, and as the queſtion aſks, how many dozen of candles, therefore 1 dozen muſt be the *middle* number of the ſecond ſtating.

Firſt, 3 pieces, each 20 ells, is 60 ells in all.

```
Yard      s.  d.     Ells.    s. d. Dozen £.  s.  d.
If 1 ———— 3 - 9 ———— 60.   If 5 - 3 be 1—14 - 1 - 3
   4        12           5    12          20
   —      ——————        ——    ——        ——————
   4        45          300   63          281
            300                            12
         ——————                         ——————
         4)13500                      63)3375(53 dozen
         ——————                          315
      12) 3375 pence                    ——————
         ——————                           225
     2|0) 281|1-3                         189
          ——————                        ——————
        £ 14 - 1 -3 Iriſh coſt             36
                                           12 in 1 doz.
                                        ——————
                                       63)432(6 candles
                                          378
                                        ——————
                                           54
```

Anſwer 53 dozen and 6, that is, 53 dozen and a half The remainder 54, and all other remainders are placed fraction-wiſe over the *diviſor*, thus, $\frac{54}{63}$. So that the true anſwer is 53 dozen and 6 candles, and 54 ſixty-three parts of another candle, or $\frac{6}{7}$, which wants but of another whole one.

Tyro.

Tyro. I underſtand it all but the $\frac{6}{7}$ths, how do you make that out.

Philo. You will ſee better when you come to *Vulgar Fractions*; for $\frac{6}{7}$ is the ſame as $\frac{54}{63}$. For divide the *Numerator* 54 by 9, and it is 6, for a *new Numerator*, and divide 63 the *Denominator* alſo by 9, and it is 7. But the anſwer without the fraction is near enough, and you muſt not mind theſe curioſities at preſent.

Queſt. 18. A Woollen-Draper bought a certain quantity of ſerge and broad-cloth for £.500: the quantity of broad-cloth he bought was 500 yards, at 12*s*. 6*d*. *per* yard. Now he bought 3 times as many yards of ſerge as he did broad-cloth. I demand how much each coſt, and what the ſerge coſt per yard?

Firſt, I find what the broad cloth comes to.

Yard.	*s.*	*d.*	*Yards.*
If 1 ——	12 -	6 ——	500
	12		

150
  500

12)75000

2|0) 6251|0 ſhillings

£312 - 10 *Anſ.* broad-cloth

	£.
Both of them coſt	500 -
Broad-cloth coſt	312 -
The ſerge coſt	187 - 10

Here you ſee the broad-cloth coſt in all £312 - 10*s*. which taken out of £.500 (what they both coſt) the ſerge then muſt coſt in all £187 - 10*s*. Now the queſtion ſays, he had 3 times as many yards of ſerge as he had of broad-cloth, which is in all 1500 yards, (that is, 3 times 500) of ſerge. Then to find how much the ſerge coſt *per* yard, I ſay,

If

Yards. £. *s.* *Yard.*
If 1500 cost 187 - 10——1
20

———

3750
12

———

First number 15|00)450|00(30 pence, or *2s. 6d.* *Ans.*
45 the serge cost *per* yard.

———

0

Tyro. Sir, I am highly obliged to you; and I perceive, that it is nothing else but considering the nature of the question, and two statings are as easy done as one. But pray, if I may be so free, is not Interest cast up, or done by the *Rule of Three*.

Philo. There is a shorter way of doing it, but it may be done very easily; as also, *exchange*, *loss*, and *gain*, *barter*, *fellowship*, and many such like rules, though they bear those different names, are all but the *Rule of Proportion:* however, to satisfy you, I will give you an example or two of each, to prepare you the better to work them.

1. An

1. *An* EXAMPLE *in* INTEREST.

Quest. 19. What is the interest of £. 360; for three years, at £. 3 ½ (or £. 3 - 10*s.*) *per cent. per annum?*

Note, Remember that *per cent.* signifies £. 100; so that in other words it is thus: What comes the interest of £. 360 to for 3 years, at £. 3 ½ for every £ 100? Say,

```
    £.     £.  s.          £.
If 100——3 - 10————————360
         20                70
         ——             ————
         70         1|00)252|00
                        —————
                    2|0)251|2   shillings
                        —————
                    £ 12—12 for 1 year Ans.
                           3 years multiply
                       ——————
                    £ 37 - 16 for 3 years.  Ans.
```

And after this manner any sum, at any rate *per cent.* may be done.

Note, When there are odd months you must allow in proportion, thus: Suppose it had been six months more, you see one year, or twelve months, is £. 12 - 12*s.* therefore, six months is half of it, *viz.* £. 6 - 6*s.* and 3 months is £ 3 - 3*s.* more.

Quest.

Quest. 19. A perſon left his niece £650, to be paid to her when ſhe came to proper age, with intereſt upon it at 4 *per cent*. Now it lay in the Executor's hands 219 days, what had ſhe got to receive principal and intereſt together?

Firſt, I find the intereſt for one year.

```
   £.  £.        £.      Days  £.      Days
If 100—4———650.  If 365 be 26———219
             4                       26
         ——————                   ——————
         £ 26|00                    1314
                                    438
                                  ——————
I cut off 2 cyphers, which is  365)5694(15 £
the ſame as divided by 100, and    365
the anſwer is £ 26 for one year's ——————
intereſt.                          2044
                                   1825
                                  ——————
Principal — £650 -                  219
129 days intereſt is 15 - 12         20
            ————————              ——————
Anſ. She has £665 - 12 to receive. )4380(12 ſhil.
                                    365
                                  ——————
                                    730
                                    730
```

2. *An* EXAMPLE *in* bartering, *or* exchanging. *See* question 17.

Quest. 21. Two merchants *A* and *B* barter, or *exchange* with one another in traffic. *A* has sugar, which he sells at £4 *per cwt.* for ready money; but in *barter* he will have £4 - 13 - 4. *B* has wine at £13 *per* hogshead: now the question is how much *B* ought to advance his wine in *barter*, to equal the advance of *A*'s sugar.

First, Sugar —— £4 - 13 - 4 in *barter*
For ready Money —— 4 -   -

£ - 13 - 4 advance

```
              £.         s.   d.      £.
Then, If 4 advance 13 - 4 ——— 13
                    12
                   ----
                   160
                    13
                   ----
                   480
                   160
                   ----
First number  4)2080
                   ----
           12)  520 pence
                   ----
          2|0)   4|3
                   ----
             £2 - 3 - 4  Ans.
```

Ans. B must advance his wine £2 - 3 - 4: So that in *barter* his wine is worth £15 - 3 - 4 *per* hogshead.

3. Ex.

3. EXAMPLES *in* profit *and* loss, *or* loss *and* gain.

Quest. 22. Bought 3 pipes of wine for 100 guineas, one of which leaked, or ran out 23 gallons; the remainder I sold at the rate of 3 half crowns *per* gallon; what was my gain or loss?

First, 1 pipe is	126 gallons.		*Gal. s. d.*	*Galls.*
	3		If 1—7-6—	355
			12	90
3 pipes is	378 gallons			
	23 leaked out		90	12)31950
	355 gallons left			2|0)266|2-6
			Sold for	£133-2-6

Made of the whole	£133 - 2 - 6
Cost 100 guineas, *viz.*	105 - -
Gain —— ——	£28 - 2 - 6 *Ans.*

Quest.

Quest. 23. What is gained *per cent.* that is, what is gained in laying out £ 100, when 1 shilling brings in 14*d.* ¾?

First	14*d.* ¾	*d.*	*d.*	£.
1 shilling	12	If 12 gain	2¾ ——	100
	———		4	20
1 shilling gains	2 ¾		——	———
			11	2000 shillings
				12
				———
				24000 pence
				11
				———
				12)264000
				———
				4(22000
				———
				12)5500
				———
				2|0) 45|8-4
				———
			Gained *Ans.*	£ 22 - 18 - 4

Quest.

Quest 24. A Gentleman that saw very plainly he could not live upon the common interest of his money, set up as common Brewer, and laid out £ 1000 for that purpose, in fitting up an office, and buying utensils; and hops, malt, excise, wages, and other charges at the year's end amounted to £ 4000 more; in which time he sold as much beer as came to £ 6875. I demand what he gained in all, and how much he gained *per cent.*

First, beer amounted to —— £ 6875
His charges are —— 5000

Gained in one year —— 1875

£. £. £.
If 5000 gain 1875 —— 100
100

*5|000) 187|500

£ 37 —— 2500 remains
20

5|000) 50|000

Ans. He gained in all
£ 1875, which is £ 37-10*s.* 10 shillings
per cent.

* *Note*, I cut off three cyphers in the *divisor*, and 3 figures in the *dividend* to answer them; then I divide 187 by 5, and it is 37, and 2 over, which, I place before the 500 that I cut off, makes 2500 remain, which I multiply by 20, and cut off 3 cyphers, and divide by 5 only, gives 10 shillings.

4. EXAMPLES *in* company *called* Fellowſhip.

Queſt. 25. Two tradeſmen or merchants, *A* and *B*, enter into partnerſhip, and purchaſe one common ſtock. *A* put in £ 1750, and *B* put in £ 1050; and they gain the firſt year of trading £ 800 clear; what is the ſhare of each, according to the ſum each advanced at firſt?

Rule. In all queſtions of this nature the rule is, Add together what every perſon firſt put into ſtock; and then ſay, as the *whole ſtock* is to the *gain* or *loſs*; ſo is what every perſon ſeparately put into his particular *gain* or *loſs*. That is, the *whole ſtock* added together is your *firſt* number; the *gain* or *loſs* your *ſecond* number; and what every man ſeparately put in is your *third* number; and the different anſwers, or *fourth* number, will be every man's proper ſhare.

The work.

A put in	—— ——	£ 1750
B put in	—— ——	1050
Whole ſtock	——	£ 2800

If 2800 gain 800——1750 *A*'s ſtock
800

28|00) 14000|00(500 £. *A*'s gain
140

00

Secondly,

£. £. £.

Secondly, If 2800 gain 800 ——— 1050 *B*'s stock.
800

28|00)8400|00(300 £. *B*'s gain.
84

00

Ans. *A*'s Share of the Gain is —— £ 500
B's Share is —— —— 300

Gain —— —— 800 proof

☞ *Note*, If the *gain* or *loss* be pounds, shillings, and pence, reduce it accordingly, and your answer will be of the same name; and if there be remains, add them together, and they will be always equal to the *divisor* if the work be right, and you must carry one to the next denomination when it happens so.

Quest. 26, A person broke, in debt for £ 1890; but he gives up all his effects to his creditors, which amounted to £. 661 - 10*s.* how much then must each creditor receive in the pound?

Note, Had each person's debt been specified, there must have been as many different statings, but in this case, but one, thus:

£. £. *s.* £.
If 1890 be 661 - 10 —— 1
20

1890)13230(7 shillings in the pound. *Ans.*
13230

0

PROOF.

Debt. £1890

7

20)1323|0

£661 - 10 effects

Now, suppose for example's sake, that any of the creditor's debt or demand was £58, what must he receive? Say, If £1 be 7 shillings, what will £58 be, and you will find it £20 - 6 for his part; and so for the others, be they more or less.

Tyro. I am extremely obliged to you, for your explaining things to me in this manner.

Philo. I have no other aim but your advantage; nor should I have been so long and particular, but that your knowledge in this *rule* is the foundation of all the higher rules, and even of the *mathematics* itself, thousands of examples more, 'tis true, might be given you; but as they depend upon what you have seen, and your constant practice, I shall give you but one more, which in some measure varies from the former ones.

Quest.

Quest. 27. A woollen-draper bought of a clothier, an equal quantity of four sorts of cloth, which cost him £ 367 - 10*s*. Some cost 18*s*. 6*d*. some 14*s*. 3*d*. some 11*s*. 6*d*. and some 4*s*. 9*d*. *per* yard: how many yards of each had he?

Rule. The method of working all sums of this sort is this: add all the different prices together into one sum, which is the *first* number; then 1 yard, or 1 *lb*. &c. will be your *second* number, and the whole sum laid out your *third* number.

The work.

	s.	d.
First, I add all the different prices thus:	18	6
	14	3
	11	6
	4	9
	£. 2 - 9	-

	£. s.	£.	£. s.
If	2 - 9	1	367 - 10
	20		20
	49 shillings		49)7350(150 yards of each. *Ans*.
			49
			245
			245
			0

Tyro. I perceive the nature of it plainly.

Philo. Indeed, *Tyro*, if you understand these examples, you are able to solve any common question relating to business.

N. B. My intent, *Tyro*, was to have given you here a notion of *timber-measure*, and how to *guage* a common cask, cooler, or piece of malt, &c. or to measure

any regular piece of ground; it being not only diverting, but alſo uſeful in the country, and very ſatisfactory to parents, when their children have ſome knowledge of theſe things. But I ſhall reſerve this for a dialogue by itſelf, and only leave you a few queſtions for practice.

QUESTIONS *to exerciſe the learner in the* Rule of Three.

28. The rents of a whole pariſh amount to £ 1750, and a rate is granted of £ 32 - 16 - 3; what is that in the pound? *Anſ.* 4*d.* $\frac{1}{2}$.

29. A bankrupt is indebted £ 2980 - 10*s.* but all his effects amount but to £ 931 - 8 - 1$\frac{1}{2}$; what have his creditors in the pound? *Anſ.* 6*s.* 3*d.*

30. Bought a lighter of coals, containing 33 chaldrons, 12 buſhels, for which I gave £ 50; what do they lie me in *per* buſhel? *Anſ.* 10*d.*

31. Bought of a Goldſmith 4 *lb.* 11 *oz.* 10 *dwts.* of plate, at 5*s.* 4*d. per* ounce; what does it come to? *Anſ.* £ 15 - 17 - 4.

32. What is the intereſt of £ 631 - 5*s.* for a year, at 3 *per. cent?* *Anſ.* £ 18 - 18 - 9.

33. What comes the commiſſion of 245 - 6*s.* to, at 2$\frac{1}{2}$ *per. cent?* *Anſ.* £ 6 - 2 - 7 $\frac{1}{4}$, $\frac{2}{5}$.

34. What muſt I give for the $\frac{6}{32}$ parts of a ſhip, that is worth £ 635 - 5*s.* *Anſ.* £ 119 - 2 - 2$\frac{1}{4}$.

35. Shipped for *Barbadoes* 500 pair of ſtockings, at 3*s.* 6*d.* per pair, and 1650 yards of baize, at 15*d. per* yard; and I have received in return 348 gallons of rum, at 6*s.* 8*d.* per gallon, and 750 *lb.* of indigo, at 1*s.* 4*d. per lb.* what remains due upon my adventure? *Anſ.* 24 - 12 - 6.

Rule. See what the ſtockings and baize come to, and add them together; then ſee what the rum and indigo both come to, and ſubtract it from the other, you will find the anſwer as above.

36. How

36. How many bricks 9 inches long, and 4 inches wide, will floor a room 18 feet square; that is, 18 feet wide, and 18 feet long?

Rule. First, find the content of the room in square feet, that is, multiply the length by the breadth; then multiply that product by 144, will bring it into square inches: this done, multiply the length of the brick by the breadth, it is 36 inches, which is your divisor. And the answer is 1296 bricks, 2 thirds.

37. How many 10 inch tiles will floor a malt kiln 16 feet long, and 14 feet wide. *Ans*. 322 $\frac{56}{100}$. *Rule*. Multiply 10 by itself, is the contents of one tile in square inches; then find the contents in the room in inches, and divide by 100, it is 322, and better than a half.

38. Bought a ton of iron, which cost £19 - 10*s*. there being in number 32 bars: I desire to know the price of each, and what they weigh one with another. *Ans*. 12*s*. 2*d*. $\frac{1}{4}$, and each bar weighs 2 *qrs*. 14 *lb*.

39. There is a cistern that holds 4 hogsheads of water, (allowing 63 gallons to the hogshead) in which are placed two pipes, the larger of which will discharge 1 $\frac{1}{2}$ gallon every minute; but if they both be set open they will discharge 2 $\frac{1}{4}$ gallons every minute: I demand in how long time the cistern will be filled by each pipe alone, and in how long when they are both set open together. *Ans*. Greater pipe will fill it in 2 hours 48 minutes; the small one in 5 hours, 36 minutes; and both in 1 hour 52 minutes.

40. There is a steeple, which stands right upon level ground, whose shadow measures 75 yards, at the same time that the shadow of a straight walking stick, which is a yard long, measures 5 feet: I demand the height of the steeple. *Ans*. 45 yards.

41. As I was beating on the forest grounds,
Up starts a hare before my two greyhounds:
The dogs be light of foot, did fairly run
Unto her 15 rods, just 21.
The distance that she started up before
Was 4 score, 16 rods just, and no more:
Now this I'd have you unto me declare,
How far they ran before they caught the hare?

Ans. 336 rods. } Hare
Dogs ran } 240 Rods

DIALOGUE VIII.

SECTION I.

The RULE of Three *Inverse*, *or* Reciprocal Proportion.

Tyro. WHAT do you mean by the *Rule of Three Inverse*?

Philo. It is often called the *Rule of Three Reverse*; that is, it is contrary to the *Rule of Three Direct*. For, as in the *Rule of Three Direct*, you multiply your *second* number by your *third*, and divide by your *first*; here you multiply your *second* by your *first*, or your *first* by your *second*, and divide by your *third*, and the *fourth number* is the answer.

Note, Your *third* and *first* number must be of the same name or denomination, as in the *Rule of Three Direct*, and the *fourth* number or answer, will be of the same name as the *second* number.

Quest.

Quest. 1. How many yards of shalloon, 3 qrs. wide, will line 4 yards of cloth, which is 5 qrs. wide? *Ans.* 6 yards $\frac{2}{3}$.

As in the *Rule of Three Direct*, so here also, the *question*, or demand lies upon the *third* number.

qrs. yards qrs.
If 5——4——3
5 first number

3)20

Ans. 6 $\frac{2}{3}$

Ans. 6 yards, and 2 remains, which I place a top of the 3, thus, $\frac{2}{3}$, and it is 2 thirds. So is the answer 6 yards $\frac{2}{3}$.

Quest. 2. If when wheat is 3 *s*. 6 *d*. a bushel, the three-penny loaf weighs 4 *lb*. 2 *oz*. what ought it to weigh when the wheat is 5 shillings a bushel? *Ans.* 2 *lb*. 11 *oz*.

s. *d.*	*lb.* *oz.*	*s.*
If 3 - 6——	4 - 2——	5
12	12 *oz.* 1 *lb.* troy	12
42 pence	50	60
	42	
	100	
	200	
	6|0)210|0	
	12)35 *oz.*	
	2 *lb.* 11 *oz.* *Ans.*	

¶ *Quest.* 3. A regiment of soldiers, consisting of 1000 men, are to have new coats, each containing 3 yards, 2 qrs. of cloth, that is, 6 qrs. wide, and they are to be lined with shalloon that is yard wide; how much cloth will it take for their cloaths, and how much shalloon will line them? *Ans.* 5250 yards.

¶ First, see how much is in their cloaths, thus:

1 coat is 3 *Yards* 2 *qrs.*
4
that is 14 *qrs.*
1000
14000 *qrs.* in all.

	qrs.	*qrs.*	*Yard.*
Then, if	6 ——	14000 ——	1 shalloon.
		6	4 *qrs.*
Third number		4)84000	
		4)21000 *qrs.*	
		5250 yards. *Ans.*	

Quest.

Quest. 4. An acre of land contains (according to the table) 40 rods in length, and 4 in breadth, what must be the length to make an acre, when the breadth is 15 rods?

Breadth *Length* *Breadth*
If 4 —— 40 —— 15
4
———
15)160(10 rods
15
———
10
5½ yards 1 rod
———
Rods Yds Feet
15)55(3 yards. *Ans*. 10 - 3 - 2
45
———
10
3 feet
———
15)30(2 feet
30
———
0

Quest. 5. If £ 100 in 12 months gain £ 4 interest, what principal will gain as much in 8 months? *Ans*. £ 150.

Months *£*. *Months*
If 12 —— 100 —— 8
12
———
8)1200
———
£ 150 *Ans*.

Quest.

Quest. 6. An army of soldiers consisting of 800, besiege a town, having with them provisions for 3 months, but as they could not take the town in that time, the General was willing to make the same provisions last 5 months, allowing each man the same daily provision: The question is, how many men must there be that the provision may last them 5 months?

Months	*Men*	*Months*		*Men*	
* If 3	—800—	5		800	for 3 months
	3			480	for 5 months
	5)2400			320	discharged
	480	*Ans*.			

The last QUESTION *varied*.

* *Note*, This question is set in words on purpose, *Tyro*, to puzzle you; but you must observe, that in all such sort of sums, the circumstance of what is done is never to be minded: for, suppose I should say thus:

If 800 men in 3 months eat 10000 penny loaves; how many will eat the same in 5 months, at that rate? *Ans*. 480 men, as before: so that the action, or circumstance of what they do is nothing, let it be eating, drinking, or any other thing.

Tyro. I understand you, Sir, and thank you for this observation; and I think you have given me sufficient examples.

Philo. All other questions are done after the same manner. If you are perfect in these, you will easily do the following ones.

QUESTIONS

QUESTIONS *to exercise this* RULE.

Quest. 7. If when wheat is sold for 3*s*. 9*d*. a bushel, the six-penny loaf weighs 5 *lb*. 10 *oz*. How much ought it to weigh when it is half a crown a bushel? *Ans*. 8 *lb*. 9 *oz*.

Quest. 8. How many yards of matting, that is 2 feet wide, will cover a room 20 feet long, and 18 feet wide. *Ans*. 180 feet, or 60 yards.

Quest. 9. How many yards of paper, 3 qrs. wide, will be sufficient to hang a room that is 24 yards round, and 4 yards high? *Ans*. 128 yards.

Tyro. These questions are sufficient, sir.
Philo. Then we will proceed to *section* 2.

SECTION II.

Of Plural Proportion, *called the* Double Rule of Three Direct.

Tyro. WHAT is *plural proportion*, or the *double rule of three direct*?

Philo. *Plural proportion* is when 5 numbers are given to find a sixth; which six numbers, or answer, is found by two statings of the *single rule of three direct*.

Tyro. This is very difficult I imagine; for how am I to know how to make the first stating.

DEFINITION *and* RULE.

Philo. In every regular sum in the *Double rule of three direct*, the first 3 numbers always shew the condition, or supposition; and the other two, that is, the two last,

last, shew the demand. The answer of the question will always be the same as the *middle* number; and the *first* and *third* must be of the same name in both statings, like as in the *single rule of three direct*.

Tyro. Cannot these questions be done at one stating?

Philo. Yes, by a rule generally called the *rule of three of* 5 *numbers*, and that with a great deal more ease, as you shall see by and by; but we will give an example here.

Quest. If £. 100 in 12 months gain £. 4 interest; what will £. 175 gain in 9 months. *Ans*. £. 5 - 5s.

```
           £.      £.        £.
First, if 100 gain 4 what—175
                           4      Here I find £175
                        ———       gains £7 in twelve
                     1|00)7|00    months. Therefore,
                        ———       I say in the second
                        £7        stating

              Mths.     £.    Mths.
              If 12—————7—————9
                        9
                       ——
                    12)63
                      £5—————3 remains
                             20
                             ——
          Same divisor    12)60
                             ——
                             5 shillings
```

Or

Or thus,

You may begin with the *second*, *third*, and *fifth* numbers,

	Months		£.		*Months*	
First, if	12	—be—	4	———	9	*Anſ.* £. 3.

	£.		£.		£.	
Then, If	100	—	3	———	175	*Anſ.* £. 5. - 5.

Note, See the proof of this queſtion in the firſt example of the next *rule inverſe*.

Queſt. 2. If 600 ſeamen in one week eat 1500*lb.* of beef; how many *lb.* will ſerve 120 ſeamen 12 weeks? *Anſ.* 3600*lb.* See *Ex.* 2, in the next rule.

All queſtions in this rule being done after the ſame manner, I ſhall leave other examples for the *rule of three of five numbers*, by which they are done with more eaſe; and ſhall only give you a notion of ſtating queſtions here and in the the *double rule of three inverſe*.

A RULE *to ſtate, or know the nature of a queſtion, and to know of the certainty of your three firſt numbers, and the name of the anſwer.*

You know there are *five* numbers given to find out a *ſixth*; mind then you carefully obſerve what name your anſwer is to be in: this being done, place your 5 numbers in two rows, *viz.* 3 in the firſt row, and 2 under them, leaving a blank ſpace under that number which is of the ſame name as your anſwer is to be in; ſo will you ſee both here, and in the next rule, the order of both theſe *double rules of three, direct and inverſe.*

Take

Take the first question, where your answer is to be in pounds, and rank the numbers thus, as the question itself runs.

If £. 100—12 months £. 4.

£. 175—9 months *

* Here you see the blank, or vacant place, is *money*; therefore, money must be your middle number of the three, and money will be the answer. *Note*, The blank must fall under the supposition, which here is £. 4.

Second QUESTION.

If 600 seamen—1 week—1500 *lb.*
120 seamen—12 weeks *

* This *third* number in the second row has a blank place under you see, according to the order of the sum; therefore, 1500 *lb.* must be your *second* number in the first stating, and your answer will also be *lbs.*

Tyro. I humbly thank you, sir.

Philo. There are many other ways of varying this rule; but this is sufficient for your purpose at present. See the *rule of* 5 *numbers*.

SEC-

SECTION III.

The Double Rule of Three Inverse, *called also* Plural Proportion.

Tyro. WHAT is the difference between this rule and the last?

Philo. A great deal; for though here be 5 numbers given to find a *sixth*, yet you will find it much more difficult to state the question here than in the other; but observe the following direction.

RULE.

1. As in the *double rule direct*, so also here are 3 numbers in the supposition, and 2 in the demand.

2. Place the numbers in order, as in the last rule, *viz.* 3 in one row, and 2 in the next, leaving that number with a blank place under, which will be the same name of your answer, observing well where the supposition lies.

3. This being done, you state the question with 3 numbers first, as before; which is wrought according to the *single rule of three inverse*.

☞ If your first stating be *direct*, the second must be *inverse*; for both are never *direct*, nor never *inverse*.

Quest.

Quest. 1. If £100 principal * in 12 months gain £4 interest, what principal will gain 5 guineas (*viz.* £5 - 5*s.*) in 9 months? *Ans.* £175 principal.

* *Note*, The word principal signifies the money put out to interest.

Here you see, there are 3 numbers in the supposition *viz* £100, 12 months, and £4; and 2 in the demand, *viz.* £5 - 5*s.* and 9 months.

Now observe, that one of these two in the demand must be the last number in the *first stating*, and the other the last in the *second stating*, and one must be always *inverse*.

		Mths.	£.	Mths.
First, *inverse*.	If	12 ——	100 ——	9

Multiply £100 by 12, and divide by 9, you have £133 $\frac{1}{3}$, *viz.* £133. 6*s.* 8*d.* Then,

		£.	£. s. d.	£. s.
Secondly, *direct*.	If	4 ——	133 - 6 - 8 ——	5 - 5.

Here I reduce the *first* and *last* numbers into shillings, the *second* into pence, and by the *rule direct*, I find the answer £175, the principal required.

2. *Or thus:*

			£.	£.	£. s.	
1.	*Direct*.	If	4 ——	100 ——	5 - 5.	*Ans.* £131-5*s.*

			Mths.	£. s.	Mths.	
2.	*Inverse*.	If	12 ——	131 - 5 ——	9.	*Ans.* £175,

as before.

Quest.

Quest. 2. If 600 seamen in 1 week eat 1500 *lb.* of beef, how long will 3600 *lb.* serve 120 men at that rate.

	Men	*Week*	*lbs.*
Here, If	600———	1 ———	1500
	3600 *lb.*	*	120 men

Men Week Men
First, Stating *Inverse*. If 600—1—120 *Ans.* 5 weeks.
Secondly, *Direct*. If 1500—5—3600 *Ans.* 12 week.

* See the second question of the last rule.

Tyro. I understand it a little, but cannot say I am perfect in it.

Philo. It is of no great signification at present; for you will see more of it in the next rule.

SECTION IV.

The Rule of Three *composed of* five Numbers.

Tyro. WHAT difference is there between this and the other two preceding rules?

Philo. Only this, that all questions in the *double rule of three direct* and *inverse*, are performed here at one stating.

Tyro. So you said before, I remember; please, therefore, to give me a rule to work it by; for, I think that is much better than the trouble of two statings.

Philo. It is so in general, but not always: however, it is easier, and better for the learner upon the whole; for here, as well as in the two former rules, the difficulty lies to know whether the question be *direct*, or *in-*

inverse, This being known, the manner of ſtating the queſtion will be eaſy, and the work as eaſy.

A ſtanding rule to ſtate queſtions.

1. As in the *ſingle rule of three direct*, ſo alſo here, the *middle number* governs the queſtion; and the anſwer will be of the ſame name.

2. Having noted which the *middle number* is, the other 4 are eaſily known; for the *two* that are in the ſuppoſition always are the *two* before the *middle numbers*; the other *two* are the demand under the *third* and *fourth* numbers.

Laſtly, Always obſerve, that your *firſt number* be of the ſame name as your *fourth*; and your *ſecond number* the ſame name as your *fifth*. This being done,

1. *The rule for all* direct *queſtions is this.*

Multiply your *firſt* number by your *ſecond*, for a *diviſor*, then multiply your *third*, *fourth*, and *fifth* together, and divide their product by the product of the *firſt* and *ſecond*, and the *quotient* will be of the ſame name as the *middle number*, be it what it will, and is the *anſwer*.

Ex.

EXAMPLE. *The first question of the two last rules.*

Quest. 1. If £100 in 12 months gain £4 interest, what will £175 gain in 9 months. *Ans.* £5 - 5*s.*

```
      £.  Mths.    £.      £.        Mths.
   If 100—12 gain 4——175——9
      12                 9
     ————              ————
     1200              1575
                          4
                     ————
                 12|00)63|00
                     ————
                     £.5 - 3 remains
                          20
                      ————
                       12)60
                      ————
                          5 shillings
```

Ans. £5 - 5*s.* See the first example of the last rules.

Quest. If 600 seamen, in 1 week, eat 1500 *lb.* of beef; how many *lb.* will serve 120 seamen 12 weeks?

```
    Men   Week    lb.    Men    Weeks
   If 600——1——1500——120——12
        1         120
     ————      ————
      600      180000
                   12
            ——————
       6|00)21600|00
            ——————
             3600 lbs.  Ans.
```

See *Ex.* 2, in the *double rule direct.*

Ano-

Another rule for the more easy stating a question.

1. *Note*, If you observe the two foregoing examples, they are stated just as the numbers follow one another in the words of the question; but it very often happens that the numbers in the sum lie contrary to the order they ought to be placed in; therefore, you should turn the question into other words and it will be easier. *Or*,

2. Set all the numbers down as they run in the question, writing the names over each; then consider what your answer is to be in, and place that down afresh for the *middle number*; then the *two* where the demand lies must be your *fourth* and *fifth*, and the other *two* your *first* and *second* numbers: the *first* must be the same as the *fourth*, and the *second* the same as the *fifth*. See the next *question*,

Quest. 3. If £4 be the wages of 8 men 10 days, what will it cost me, or what must be the wages of 32 men 12 days? *Ans.* £19 - 4*s.*

Here, as before directed, I place the numbers after one another, thus:

	£.	Men	Days	Men	Days
If	4——	8——	10——	32——	12. wrong stated.

Now, here the *middle number* is days, but should be money, because the question says, what will it cost me, for 32 men 12 days; therefore money must be my middle number, and then, *Tyro*, the true *stating* will appear easy as follows.

	Men	Days	£.	Men	Days
Thus *,	8——	10——	4——	32——	12. *Ans.* £19 - 4*s*

Now here, *Tyro*, the stating runs like the words of the sum, as follows, which is very natural.

* If 8 men in 10 days cost me £4 for wages, what will 32 men cost me in 12 days, at that rate? *Ans.* as before.

Quest.

Quest. 4. (*A proof of the first question in the last rule*) A person put out 175 pounds to receive interest, and when it had continued 9 months, he received principal and interest together £180 - 5*s.* I demand at what rate *per cent. per annum*, he put his money out at? *Ans.* £4 *per cent. per ann.*

Here the words of the question are not like the *stating*. You must observe then to proceed as under, in all questions of this nature.

Rule. Take the principal from the interest and principal together, the remainder will be the interest for the time it continued, and then the stating will follow.

Thus, Principal and interest		£180 - 5*s.*
Principal put out	——	175 -
Interest for 9 months	——	£5 - 5*s.*

	£.	*Mths*		£. *s.*		£.	*Mths.*
If	175	—9	gain	5 - 5	interest, what will	100	in 12

Here, according to the rule, bring your *middle number* into shillings; then multiply it by your *fourth* and *fifth*, for a *dividend*; this done, multiply your *first* and *second* together for a *divisor*, and the quotient will be 80 shillings, or £4, the rate he put the money out at.

Tyro. I heartily thank you; and I find by this I shall be able to answer Mr. *Cocker*'s last question in the *double rule of three direct*, which before always appeared very difficult to me.

Philo. You say very right, *Tyro*, for the question is done after the very same manner; therefore, shall give you only two or three examples in the *double rule of three inverse*, and leave you to practice the other examples.

QUES-

QUESTIONS *Inverse*

2. The rule for *questions inverse* is as follows: the 3 numbers belonging to the supposition must be your *first*, *second*, and *third* number, and the demand is your *fourth* and *fifth* number. These being observed, the rule is, multiply your *second*, *third*, and *fourth* together for a *dividend*, and your *first* and *fifth* together for a *divisor*, and the *quotient* is the same answer as your *middle number*.

Quest. 1, (*A proof to the very last question, and of question* 1 *in the last rule*) if £100 in 12 months gain £4 interest, what principal will 5 guineas gain in 9 months?

Here, as the principal is to be the answer, so the principal (*viz.* £100) must be your *middle number*,

	£.	*Months*	*£.*		*£.*	*s.*	*Months*	
If	4 —	12 be	100 principal, what		5 -	5 be	9	*Ans.* 175.

I state the question according to the first standing rule, making my *first* and *fourth*, and my *second* and *fifth* alike; this done, I bring my *first* number into shillings (*viz.* 80) because the *fourth* must be brought into shillings: this being done, I multiply the *second*, *third*, and *fourth* together for a *dividend*, and the *first* and *fifth* together for a *divisor*, and the *quotient* is £175, the principal required.

Quest. 2. If 600 seamen in 1 week eat 1500 *lb.* of beef, how long will 3600 last 120 seamen?

	lb.	*Men*	*Week*	*lb.*	*Men.*
If	1500 ——	600 ——	1 ——	3600 ——	120

Here I multiply the *second*, *third*, and *fourth* together for a *dividend*, which is 2160000, and my *first* and *fifth* for a *divisor*, which is 180000, and the *quotient* is 12 Weeks

Quest.

Quest. 3. If it cost £240 for 60 servants in 8 weeks, how long will £140 serve 8 servants? *Ans.* 35 weeks.

Tyro. Sir, these examples are sufficient, I think.

Philo. If you are well grounded in the nature of the rule, that is enough; for you may set yourself sums at any time by way of exercise. And now, *Tyro*, I will give you a notion of *tare* and *tret*.

SECTION V.

TARE *and* TRET.

Tyro. WHAT do you mean by *tare* and *tret*?

Philo. *Tare* and *tret* are allowances made in buying and selling commodities that are liable to loss or waste.

Tyro. What are the general names belonging to this rule.

Philo. They are 6, *viz.* *gross*, *tare*, *tret*, *suttle*, *cloff*, and *neat*.

Tyro. Please to explain these in their order.

Philo. *First*, *Gross* signifies the whole weight of any commodity, or parcel of goods, bag, or box, or cask, and all included thus: suppose I had a box of tea, bag of spice, or cask of oil, which weighs 120 *lb.* I say it weighs 120 *lb.* *gross*.

Secondly, *Tare* is an allowance made for the weight of the box, bag, or cask, and what hangs about it, and is to be taken out of the *gross*, and then the remainder is called the *neat*, or *clear weight* of the commodity: thus, suppose the foregoing box, bag, or cask, after the goods are out, should weigh 16 *lb.* then

I ſay, it has 16 *lb.* *tare*, which I ſubtract out of the *groſs* 120 *lb.* there remains 104 *lb.* *neat*, thus:

Bought a box of tea,	*Groſs*	120 *lb.*
	Tare	16 *lb.*
	Neat	104 *lb.*

Thirdly, *Tret* is an allowance of 4 *lb.* for every 104; that is, 1 *lb.* is allowed for every 26 *lb.* for ſuch commodities as are liable to waſte, moths, duſt, *&c.*

Note, When there is *tret* in the ſum, then after the *tare* is taken from the *groſs*, the remainder is called *ſuttle*, inſtead of *neat.*

Note 2. To find the *tret* pounds, divide the *ſuttle* pounds by 26, and the *quotient* in the *tret*, which ſubtracted from the *ſuttle* gives the *neat*.

Thus, A box of tea,	*Groſs*	120 *lb.*
	Tare	16
26)104(4 *tret*		
104	*Suttle*	104 *lb.* which divide by 26
	Tret	4 *lb.*
0		
	Neat	100 *lb.*

Fourthly, *Suttle* pounds are ſo called (as was ſaid before) when there is *tret* in the ſum; for when there is no *tret* there is no *ſuttle*, and the *tret* pounds are always ſubtracted out of them, as in the above.

Fifthly, *Cloff* is an allowance of 2 *lb.* for every 3 *Cwt.* for the turn of the ſcale, and for ſmall draughts; but this is allowed only in ſome particular things, and caſes.

Sixthly, *Neat* weight is the *clear* weight of the commodity after all the allowances are made.

Tyro.

Tyro. Sir, I underſtand you very well; but if I remember, you told me, (in *page* 164) you would ſhew me a ſhorter method to reduce *Cwts.* and *qrs.* into *lbs.* than you did in *page* 121.

Philo. I did ſo; and will now ſhew you all the ways, and leave you to take your choice.

To reduce hundreds *and* quarters *into* pounds.

EXAMPLE I. Reduce 17 *C.* 3 *qrs.* 15 *lb.* into *lbs.*

	C. qrs. lb.		C. qrs. lb.
Firſt Method	17 - 3 - 15	*Second method*	17 - 3 - 15*
	4		17
	71		17
Common way	28		17
	573	Add	99 for the 3 *qrs.*
	143		15 *lbs.*
	Anſ. 2003 *lbs.*	*Anſ.*	2003 *lb.* See *p.* 121.

* *Note,* In this method you muſt always ſet the *C's* down 4 times, *viz.* twice under one another, and the other two, each one place more towards the left-hand; then count how many *lbs* there are in the odd *qrs.* and *lbs.* (which here are 99) and always place them under the *units*, *tens*, as above, and adding them together, you have the *anſwer*.

Note, This method is ſhorter than the *firſt*, and more particular when there are many figures in the given *hundreds*.

Third method. Multiply by 112, and take in the odd pounds.

C.	*qrs.*	*lb.*		*C.*	*lbs.*
17 -	3 -	15	is	17 and	99
				112 *	
				Anſ. 2003	

* Here I multiply by 112 in 1 line, thus: I ſay, 12 times 7 is 84, and 9 odd *lbs.* make 93, 3, and I carry 9; then 12 times 1 is 12, and 9 I carried is 21, and the other 9 I take in alſo is 30, that is 0, and I carry 3. *Laſtly*, I ſay, once 17 is 17, and 3 I carried is 20, which I place by the ſide of the other 2 figures, and it is done.

This is alſo ſhorter than the *firſt method*, but ſomething more difficult till you learn to multiply by 112 very well, as you will ſee in *Ex.* 2.

☞ *Fourth method.* The beſt way (in my opinion) is this laſt method, which is this: multiply the *C*'s by 12 only, always remembering to place the two firſt figures out towards the right-hand; then ſet the odd pounds under, and add them together as they ſtand, it is done.

	C.	*qrs.*	*lbs.*		*C.* *lbs.*
Thus,	17 -	3 -	15	is	17 - 99 ‡
					204
					99
				Anſ.	2003 *lbs.* as before.

There is no occaſion to ſet down the 12 under the 17, when you multiply; for it is eaſy to remember you multiply by 12. I ſay then 12 times 7 is 84, (which 4 I place 2 figures towards the right hand of the 17) that

that is 4 and I go 8; then 12 times 1 is 12, and 8 is 20, which I place to the left of the 4, ſo that the 2 falls under the 7; then I ſet down the 99 odd pounds, and add them together, they make 2003 *lb.*

‡ *Note*, But you may do it yet ſhorter by uſe, and full as eaſy, and that is by taking in the 99 as you multiply, which will ſave the trouble of adding it afterwards.

	C.	lb.
Thus,	17 -	99
		303
		2003

Here I ſay, 12 times 7 is 84, and 9 is 93, 3, and I carry 9; 12 times 1 is 12, and 9 I carry is 21, and the laſt 9 belonging to the 99 makes 30.

Now, the reaſon of this will appear plain, if you conſider that 17 *C.* is 1700 *lb.* and 17 times 12 *lb.* therefore, I only ſet down 1700 *lb.* and add 12 times 17 to it.

Thus, 17 *C.* is —— 1700
More, 12 times 17 is 204 add

Anſ. 1904 *lb.* in 17 *C.*

1700 — 99 *lb.*
So alſo 303 is 12 times 17, and 99.

2003 as before

☞ *Note*, Or you may do it ſhorter yet, only by putting or joining the odd *lbs.* by the ſide of the *C's* (making a dot between them for diſtinction ſake) then multiply the *C's* only by 12, placing 2 figures out from them,

them, as before directed, and you have the anſwer, which will ſave the trouble of taking in.

C.
Thus, 17 - 99 I ſet down 17.99
204

Anſ. 2003*lb.*

Here I multiply 17 only by 12, always placing the two firſt figures to the right-hand: then add the two lines together.

Example 2. In 47 *C.* 1 *qr.* 26 *lb. neat*, how many *lbs.*

C. *qr.* *lb.* *C.* *lb.*
Here 47 - 1 - 26 is 47.54 multiply 47 only by 12.
47 multiplied by 12 is 564

5318 *Anſ.*

Example 3. In 86 *C.* 1 *qr.* 17 *lb. neat*, how many *lbs.*

C. *qr.* *lb.* *C.* *lb.*
Here 86 - 1 - 17 is 86 45
86 multiplied by 12 is 1032

9677 *Anſ.*

Here I join the odd 45 *lb.* cloſe to the 86 *C.* and then multiply 86 only by 12, placing two figures out as before.

Tyro. Very well, ſir, I underſtand what you have ſhewn me; but ſuppoſe the odd pounds amount to three figures, may I join them to the *C*'s then?

Philo. No, only 2 of them; and place the other figure (which will always be 1) under the units place of the *C*'s, as in the following example.

Ex-

Example 4. Reduce 89 *C.* 3 *qrs.* 21 *lb.* *neat* into *lbs.*

That is, 89 C. and 105 *lb.* Thus, 89
3 *qrs.* 21 *lb.* is ——— 105 *lb.*
89 multiplied by 12 is ——— 1068

Anf. 10073 *lb.*

Here 3 *qrs.* 21 *lbs.* is 105 *lbs.* and becauſe there are three figures, I place the 1 under the 9, and thus for all ſuch examples.

Now, *Tyro*, I will prove this laſt example by the other three methods, and leave you to take your choice as practice or fancy direct.

C. qrs. lb.	*Secondly*, C. lbs.	*Thirdly*, C. lbs.
89 - 3 - 21	89 - 105	89 - 105*
4	89	112
359	89	10073 *lb.*
28	89	
2873	105	
720	10073 *lb.*	
10073 *lb.* common		

* This third method is certainly very ſhort, if once you can multiply well by 112: for, obſerve, I ſay 12 times 9 is 108, (and 5 the odd *lbs.*) is 113; 3, and I carry 11; then 12 times 8 is 96, and 11 is 107, and the 10 *lb.* is 117, that is 7, and I carry 11; *laſtly*, I ſay *once* 89 is 89, and 11 I carried is 100.

Tyro. Sir, I heartily thank you, I ſee now how it is done.

Philo. Then I will give you one example at large.

Example 5. Bought 3 hogsheads of sugar, as under.

		C.	qrs.	lb.
Grofs No	1—	11	2	15
	2—	15	1	17
	3—	17	3	14
Grofs		44	3	18
Tare		1	1	2
Suttle		43	2	16

Tare of all 1 *C.* 1 *qr.* 2 *lbs.* *Tret* 4 *lb.* for every 104, at 56 shillings a *C. neat*, what come they to?

Suttle	4888 *lbs.*
Tret	188 *lbs.*
Neat	4700 *lbs.*

lb.
26)4888)188 *tret*
26
—
228
208
——
208
208

Then, if 1 *C. viz.* 112 *lb.* cost 2 *l.* 16 *s.* what cost 4700 *lbs. neat?* *Anf.* £117 - 10*s.*

Example 6. Bought 3 chests of tea, each weighing *grofs*, 1 C. 3 *qrs.* 12 *lbs.* *tare* of each 1 *qr.* 5 *lb.* at 4*s.* 9*d.* a *lb. neat*; what come they to?

Here being no *tret*, the *tare* taken out of the *grofs* the remainder is *neat*. *Anf.* £131 - 6 - 9.

Tyro. I am highly obliged to you, sir; and am now ready for the next rule, if you please,

Philo. The next thing I shall instruct you in is the rule of *practice*.

DIA-

DIALOGUE IX.

SECTION I.

Of PRACTICE.

Tyro. WHAT do you mean by *practice?*

Philo. *Practice* is only *division of money*, and very much resembles *reduction ascending*; for here you divide by as many of the *less* as make one of the *greater*. It is a short contracted way of the *rule of three direct*, and saves the trouble of stating questions, by dividing by the parts contained in the whole.

Tyro. Are not *vulgar fractions* often taught before *practice?*

Philo. They are; but there is no necessity for it; if you can read a fraction that is enough. Thus, $\frac{2}{7}$, $\frac{3}{4}$, $\frac{1}{8}$, or $\frac{1}{12}$, are thus read, 2 *sevenths*, 3 *fourths*, 1 *eighth*, and 1 *twelfth*.

Tyro. I know this very well sir.

Philo. Then you may proceed to work this rule as soon as you please; but it will be necessary to learn the following tables by heart.

TABLE 1.

Even parts of a shilling.

Pence		
6	is	$\frac{1}{2}$ of a shill.
4		$\frac{1}{3}$
3		$\frac{1}{4}$
2		$\frac{1}{6}$
$1\frac{1}{2}$ *qrs.*		$\frac{1}{8}$
1		$\frac{1}{12}$
$\frac{3}{4}$		$\frac{1}{16}$
$\frac{1}{2}$		$\frac{1}{24}$
$\frac{1}{4}$		$\frac{1}{48}$

TABLE 2.

Even parts of a £. Sterling.

Shillings		
10	is	$\frac{1}{2}$ of a £.
6-8*d.*		$\frac{1}{3}$
5		$\frac{1}{4}$
4		$\frac{1}{5}$
3-4*d.*		$\frac{1}{6}$
2-6*d.*		$\frac{1}{8}$
2		$\frac{1}{10}$
1-8*d.*		$\frac{1}{12}$
1-3*d.*		$\frac{1}{16}$
1		$\frac{1}{20}$

Tyro. I will learn these tables as fast as I conveniently can.

Philo. Very well; then I shall proceed to give you examples under the following *cases.*

CASE 1.

When the sum is given in farthings, that is 1, 2, or 3 farthings *per lb.* or yard; then only bring these farthings into *pence, shillings,* and *pounds,* and the work is done.

$\frac{1}{4}$	$\frac{1}{4}$	1753 *lb.* at $\frac{1}{4}$ a *lb.*
1*d*	$\frac{1}{12}$	438-$\frac{1}{4}$
1*s.*	2|0	3|6 - 6*d.*
		£1-16 6$\frac{1}{4}$ *Ans.*

Here 1753 *lbs.* at a farthing a *lb.* is 1753 farthings; therefore, I only divide by 4, by 12, and by 20, and have £1-16-6$\frac{1}{4}$ for answer.

Any

$\frac{1}{2}$*ny*	$\frac{1}{2}$	1753 *lb.* at $\frac{1}{2}$*ny* a *lb.*
1*d.*	$\frac{1}{12}$	876-$\frac{1}{2}$
1*s.*	2|0	7|3-
		£3-13-0$\frac{1}{2}$ *Anſ.*

Here 1753 is 1753 halfpence. I divide therefore by 2 to bring them into pence, and by 12, and 20, to bring them into £'s, and the anſwer is £3 - 13 - 0$\frac{1}{2}$, which is juſt double the firſt ſum, at a farthing a *lb.* and is a proof to it.

*$\frac{3}{4}$	$\frac{1}{8}$	1753 *lb.* at $\frac{3}{4}$ *per lb.*
6*d.*	$\frac{1}{2}$	219-$\frac{3}{4}$
1*s.*	2|0	10|9-6
		£5-9-6$\frac{3}{4}$ *Anſ.*

Here I ſay 3 farthings is the 1 eighth of 6 pence, and dividing by 8, I have 219 ſix-pences, and 3 farthings over; then I divide by 2 (becauſe 6*d.* is $\frac{1}{2}$ a ſhilling) and have 109 ſhillings, and 6*d.* over; then I divide by 20, to bring them into £'s, and have £5-9-6$\frac{3}{4}$.

Note, If you add the anſwers of the *firſt* and *ſecond* ſum together, you will find the amount to £5 - 9 - 6$\frac{3}{4}$ which proves the work.

* There is another way to do this Multiply 1753 by 3 farthings, and the product is farthings; than divide by 4, 12, and 20.

	1753 *lb.* at $\frac{3}{4}$ *per lb.*
	3
4	5259
12	1314 - $\frac{3}{4}$
2[0	10[9 - 6
	£5-9-6$\frac{3}{4}$ *Ans.*

d.		*Yds.*
1	$\frac{1}{12}$	1753‡ at 1*d. per* yd.
	2[0	14[6 - 1
		£7-6-1 *Ans.*

‡ *Note*, I make use of the same number, 1753, because you may prove the work of one sum by another, by adding the products of any two together, thus; the product of 3 farthings and 1 farthing will prove the last sum of a penny.

CASE 2.

When the given price is pence and farthings, then take the parts for the pence first, and then the parts for the farthings.

d.		*Ells* *d.*
1	$\frac{1}{12}$	1753 at 1$\frac{1}{4}$ *per* ell.
$\frac{1}{4}$	$\frac{1}{4}$	146 - 1*d.*
		36 - 6$\frac{1}{4}$ †
	2[0	18[2 - 7$\frac{1}{4}$
		£9 - 2 - 7$\frac{1}{4}$ *Ans.*

† Here, I say 1 penny is the $\frac{1}{12}$ of a shilling; then I say 1 farthing is the $\frac{1}{4}$ of a penny, and dividing 146 shillings by 4, I have 36 shillings, and 2 remains, which is 2 shillings; then I say, the 4th. of 2 shillings is 6*d.* and the 4th of a penny is $\frac{1}{4}$; then I add them both together, which make 182*s.* 7*d.* $\frac{1}{4}$, and dividing by 20, I have £9-2-7$\frac{1}{4}$ for *answer*

☞ Always remember that what remains is shillings and pence, and you must also divide such shillings

ſhillings by the ſame figure, and you have the true anſwer.

d.		*lb.*
$1\frac{1}{2}$	$\frac{1}{8}$	1753 at $1\frac{1}{2}$ *per lb.*
	2|0	21|9 - $1\frac{1}{2}$
		£10-19-1 $\frac{1}{2}$ *Anſ.*

* This you ſee is juſt double the ſum of 3 farthings.

d.		*lb.*
$1\frac{1}{2}$	$\frac{1}{8}$	1753 at $1\frac{3}{4}$
$\frac{1}{4}$	$\frac{1}{6}$	219 - $1\frac{1}{2}$
		36 - $6\frac{1}{4}$*
	2|0	25|5 - $7\frac{3}{4}$
		£12-15-$7\frac{3}{4}$

* *Note,* Here I ſay 3 half-pence is the $\frac{1}{8}$th (as before) then I ſay, 1 farthing is the $\frac{1}{6}$th of 3 halfpence, and dividing by 6, I have 36, and 3 remains, which 3 I call 3 ſhillings; then I ſay, the 6th part of 3 ſhillings is 6 pence, and the 6th part of $1\frac{1}{2}$ is $\frac{1}{4}$. Thus you ſee I take the parts of the remainder as well as the parts of the whole.

Pray obſerve, *Tyro*, and look well at the example; for I ſhall now leave you to yourſelf for a while.

		Yards
2*d*	$\frac{1}{6}$	1753 at 2*d.* *per* yd.
	2|0	29|2 - 2*d.*
		£14-12-2 *Anſ.*

		lb.
2*d*	$\frac{1}{6}$	1753 at $2\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{8}$	292 - 2*d.*
		36 - $6\frac{1}{4}$
	2|0	32|8 - $8\frac{1}{4}$
		£16-8-$8\frac{1}{4}$ *Anſ.*

		oz. *d.*
2*d*	$\frac{1}{6}$	1753 at $2\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{4}$	292 - 2*d.*
		73 - $0\frac{1}{2}$
	2|0	36|5 - $2\frac{1}{2}$
		£18-5-$2\frac{1}{2}$

Ell

Ells
2*d* | $\frac{1}{6}$ | 1753 at 2*d.* $\frac{3}{4}$

$\frac{1}{2}$ $\frac{1}{4}$ | $\frac{1}{4}$ $\frac{1}{2}$ | 292 - 2*d.*
73 - 0$\frac{1}{2}$
36 - 6$\frac{1}{4}$

2|0 | 401|1 - 8$\frac{3}{4}$

£20-1-8$\frac{3}{4}$ *Ans.*

lb.
3*d* | $\frac{1}{4}$ | 1753 at 3*d.*

2|0 | 431|8 - 3

£21-18-3 *Ans.*

lb.
3*d* | $\frac{1}{4}$ | 1753 at 3*d.* $\frac{1}{4}$

$\frac{1}{4}$ | $\frac{1}{12}$ | 438 - 3
36 - 6$\frac{1}{4}$

2|0 | 471|4 - 9$\frac{1}{4}$

£23-14-9$\frac{1}{4}$ *Ans.*

lb. *d.*
3*d* | $\frac{1}{4}$ | 1753 at 3 $\frac{1}{2}$ *lb.*

$\frac{1}{2}$ | $\frac{1}{6}$ | 438 - 3*d.*
73 - 0$\frac{1}{2}$

2|0 | 511|1 - 3$\frac{1}{2}$

£25-11-3$\frac{1}{2}$

lb. *d.*
3*d* | $\frac{1}{4}$ | 1753 at 3 $\frac{3}{4}$ *lb.*

$\frac{3}{4}$ | $\frac{1}{4}$ | 438 - 3*d.*
109 - 6$\frac{3}{4}$

2|0 | 547 - 9$\frac{3}{4}$

£24-7-9$\frac{3}{4}$ *Ans.*

Dozen
4*d* | $\frac{1}{3}$ | 1753 at 4*d.*

2|0 | 58|4 - 4

£29-4-4 *Ans.*

lb. *d.*
3*d* | $\frac{1}{4}$ | 1753 at 4$\frac{1}{4}$ *lb.*

1*d* $\frac{1}{4}$ | $\frac{1}{3}$ $\frac{1}{4}$ | 438 - 3
146 - 1
36 - 6$\frac{1}{4}$

2|0 | 62|0 - 10$\frac{1}{4}$

£31-0-10$\frac{1}{4}$ *Ans.*

lb. *d.*
4*d* | $\frac{1}{3}$ | 1753 at 4$\frac{1}{2}$ *lb.*

$\frac{1}{2}$ | $\frac{1}{8}$ | 584 - 4
73 - 0$\frac{1}{2}$

2|0 | 65|7 - 4$\frac{1}{2}$

£32-17-4$\frac{1}{2}$

4d	$\frac{1}{3}$	1753 *lb.* at 4*d.* $\frac{3}{4}$ *lb.*
$\frac{1}{2}$	$\frac{1}{8}$	584 - 4*d*
$\frac{1}{4}$	$\frac{1}{2}$	73 - 0$\frac{1}{2}$
		36 - 6$\frac{1}{4}$
	2|0	69|3-10$\frac{3}{4}$
		£34-13-10$\frac{3}{4}$

4d	$\frac{1}{3}$	1753 *lb.* at 5*d lb.*
1d	$\frac{1}{4}$	584 - 4
		146 - 1
	2|0	73|0- 5
		£36-10-5 *Ans.*

		Ells
4d	$\frac{1}{3}$	1753 at 5*d.* $\frac{1}{4}$
1d	$\frac{1}{4}$	584 - 4
$\frac{1}{4}$	$\frac{1}{4}$	146 - 1
		36 - 6$\frac{1}{4}$
	2|0	76|6 - 11$\frac{1}{4}$
		£38-6-11$\frac{1}{4}$ *Ans.*

4d	$\frac{1}{3}$	1753 *lb.* at 5*d.* $\frac{1}{2}$ *lb.*
1d	$\frac{1}{4}$	584 - 4
$\frac{1}{2}$	$\frac{1}{2}$	146 - 1
		73 - 0$\frac{1}{2}$
	2|0	80|3 - 5$\frac{1}{2}$
		£40-3-5$\frac{1}{2}$ *Ans.* *

* Or you may take the parts thus, *viz.* 4*d.* is $\frac{1}{3}$ of a shilling, as before.

Or thus,

4d	$\frac{1}{3}$	1753 *lb.* at 5 $\frac{1}{2}$ *lb.*
1$\frac{1}{2}$	*$\frac{1}{8}$	584 - 4*d*
		219 - 1$\frac{1}{2}$
	2|0	80|3 - 5$\frac{1}{2}$
		£40 - 3 5$\frac{1}{2}$ *Ans.*

* Here I say, 1$\frac{1}{2}$ is $\frac{1}{8}$ of a shilling, and take it out of 1753, the top-line, which answers the same end as the last work

		lb. *d.*
4d	$\frac{1}{3}$	1753 at 5 $\frac{3}{4}$
1$\frac{1}{2}$	$\frac{1}{8}$	584 - 4*d.*
$\frac{1}{4}$	$\frac{1}{6}$	219 - 1$\frac{1}{2}$
		36 - 6$\frac{1}{4}$
	2|0	83|9 - 11$\frac{3}{4}$
		£41-19-11$\frac{3}{4}$ *Ans.*

6d	$\frac{1}{2}$	1753 *lb.* at 6*d. lb.*
	2|0	87|6-6
		£43-16-6 *Ans.*

Yards

		Yards
3*d*	$\frac{1}{4}$	1753 at 6*d*. $\frac{1}{4}$ yard
d.		
3*	$\frac{1}{4}$	438 - 3
$\frac{1}{4}$	$\frac{1}{12}$	438 - 3
		36 - 6$\frac{1}{4}$
	2|0	91|13 - 0$\frac{1}{4}$
		£45-13-0$\frac{1}{4}$ *Ans.*

* Here I say 3 pence is the $\frac{1}{4}$ of a shilling in the second line as well as in the first, because then $\frac{1}{4}$ will be the $\frac{1}{12}$; whereas, if I was to say 6*d*. is the $\frac{1}{2}$ of a shilling, then I must say $\frac{1}{4}$ is the $\frac{1}{24}$ of 6 pence: now 24 is not so easy to divide by as 12 is; so by bringing sixpence into 2 parts, the work is a great deal more easy and natural.

		Bushels *d.*
6*d*	$\frac{1}{2}$	1753 6$\frac{1}{2}$ *bushel*
$\frac{1}{2}$	$\frac{1}{12}$	876 - 6
		73 - 0$\frac{1}{2}$
	2|0	94|19 - 6$\frac{1}{2}$
		£47-9-6$\frac{1}{2}$ *Ans.*

		oz.
6*d*	$\frac{1}{2}$	1753 at 6$\frac{3}{4}$ *oz.*
$\frac{3}{4}$	$\frac{1}{8}$	876 - 6
		109 - 6$\frac{3}{4}$
	2|0	98|16 0$\frac{3}{4}$
		£49-6-0$\frac{3}{4}$ *Ans.*

		lb. *d.*
6*d*	$\frac{1}{2}$	1753 at 7 *lb.*
1*d*	$\frac{1}{6}$	876 - 6*d.*
		146 - 1*d.*
	2|0	102|12 - 7
		£51 - 2 - 7 *Ans.*

		Gallons
6*d*	$\frac{1}{2}$	1753 at 7$\frac{1}{4}$ *gallon*
1*d*	$\frac{1}{6}$	876 - 6*d.*
$\frac{1}{4}$	$\frac{1}{4}$	146 - 1
		36 - 6$\frac{1}{4}$
	2|0	105|19 - 1$\frac{1}{4}$
		£52 19 1$\frac{1}{4}$ *Ans.*

6*d*	$\frac{1}{2}$	1753 *lb.* at 7$\frac{1}{2}$ *lb.*
1$\frac{1}{2}$	$\frac{1}{4}$	876 - 6
		219 - 1$\frac{1}{2}$
	2|0	109|15 - 7$\frac{1}{2}$
		£54-15-7-$\frac{1}{2}$ *Ans.*

Ells

4d	$\frac{1}{3}$	1753 *ells* at $7\frac{3}{4}$
3d	$\frac{1}{4}$	584 - 4
$\frac{3}{4}$	$\frac{1}{4}$	438 - 3
		109 - $6\frac{3}{4}$
	2|0	113|2 - $1\frac{3}{4}$
		£56-12-$1\frac{3}{4}$ *Ans.*

Or you may say 6*d.* is $\frac{1}{2}$, 1*d.* is $\frac{1}{6}$, and $\frac{3}{4}$ is the $\frac{1}{8}$ of 6 pence.

4d	$\frac{1}{3}$	1753 *yards* at 8*d.*
4d	$\frac{1}{3}$	584 - 4
		584 - 4
	2|0	116|8 - 8
		£58-8-8 *Ans.*

Or you may say 6*d.* is $\frac{1}{2}$, and 2*d.* is $\frac{1}{3}$ of 6 pence; or as 8*d.* is $\frac{2}{3}$ of a shilling, you may multiply by 3, and divide by 2.

6d	$\frac{1}{2}$	1753 *lb.* at $8\frac{1}{4}$
2d	$\frac{1}{3}$	876 - 6
$\frac{1}{4}$	$\frac{1}{8}$	292 - 2
		36 - $6\frac{1}{4}$
	2|0	120|5 - $2\frac{1}{4}$
		£60-5-$2\frac{1}{4}$

4d	$\frac{1}{3}$	1753 *pair* at $8\frac{1}{2}$
4d	$\frac{1}{3}$	584 - 4
$\frac{1}{2}$	$\frac{1}{8}$	584 - 4
		73 - $0\frac{1}{2}$
	2|0	124|1 - $8\frac{1}{2}$
		£62-1-$8\frac{1}{2}$ *Ans.*

4d	$\frac{1}{3}$	1753 *ells* at 8*d.* $\frac{3}{4}$
4d	$\frac{1}{3}$	584 - 4
$\frac{1}{2}$	$\frac{1}{8}$	584 - 4
$\frac{1}{4}$	$\frac{1}{2}$	73 - $0\frac{1}{2}$
		36 - $6\frac{1}{4}$
	2|0	127|8 - $2\frac{3}{4}$
		£63-18-$2\frac{3}{4}$ *Ans.*

6d	$\frac{1}{2}$	1753 *lb.* at 9*d.*
3d	$\frac{1}{2}$	876 - 6
		438 - 3
	2|0	131|4 - 9
		£65-14-9 *Ans.*

6d	$\frac{1}{2}$	1753 *yards* at $9\frac{1}{4}$
3d	$\frac{1}{2}$	876 - 6
$\frac{1}{4}$	$\frac{1}{12}$	438 - 3
		36 - $6\frac{1}{4}$
	2|0	135|1 - $3\frac{1}{4}$
		£67-11-$3\frac{1}{4}$ *Ans.*

Ells

6d	$\frac{1}{2}$	1753 *ells* at $9\frac{1}{2}$
3d	$\frac{1}{2}$	876 - 6
$\frac{1}{2}$	$\frac{1}{6}$	438 - 3
		73 - $0\frac{1}{2}$
	2|0	138|7 - $9\frac{1}{2}$
		£69-7-$9\frac{1}{2}$ *Ans.*

6d	$\frac{1}{2}$	1753 *lb.* at $9\frac{3}{4}$
3d	$\frac{1}{2}$	876 - 6
$\frac{3}{4}$	$\frac{1}{4}$	438 - 3
		109 - $6\frac{3}{4}$
	2|0	142|4 - $3\frac{3}{4}$
		£71-4-$3\frac{3}{4}$ *Ans.*

6d	$\frac{1}{2}$	1753 *ells* at 10*d.*
4d	$\frac{1}{3}$	876 - 6
		584 - 4
	2|0	146|0- 10
		£73-0 10 *Ans.*

6d	$\frac{1}{2}$	1753 *oz.* at $10\frac{1}{4}$
3d	$\frac{1}{2}$	876 - 6
1d	$\frac{1}{3}$	438 - 3
$\frac{1}{4}$	$\frac{1}{4}$	146 - 1
		36 - $6\frac{1}{4}$
	2|0	149|7- $4\frac{1}{4}$
		£74-17-$4\frac{1}{4}$ *Ans.*

6d	$\frac{1}{2}$	1753 *lb.* at $10\frac{1}{2}$
3d	$\frac{1}{2}$	876 - 6
$1\frac{1}{2}$	$\frac{1}{2}$	438 - 3
		219 - $1\frac{1}{2}$
	2|0	153|3 - $10\frac{1}{2}$
		£76-13-$10\frac{1}{2}$ *Ans.*

6d	$\frac{1}{2}$	1753 *oz.* at $10\frac{3}{4}$
3d	$\frac{1}{2}$	876 - 6
$1\frac{1}{2}$	$\frac{1}{2}$	438 - 3
$\frac{1}{4}$	$\frac{1}{6}$	219 - $1\frac{1}{2}$
		36 - $6\frac{1}{4}$
	2|0	157|0 - $4\frac{3}{4}$
		£78-10-$4\frac{3}{4}$ *Ans.*

6d	$\frac{1}{2}$	1753 *oz.* at 11*d.*
4d	$\frac{1}{3}$	876 - 6
1d	$\frac{1}{4}$	584 - 4
		146 - 1
	2|0	160|6- 11
		£80-6-11 *Ans.*

6d	$\frac{1}{2}$	1753 *ells* at $11\frac{1}{4}$
4d	$\frac{1}{3}$	876 - 6
1d	$\frac{1}{4}$	584 - 4
$\frac{1}{4}$	$\frac{1}{4}$	146 - 1
		36 - $6\frac{1}{4}$
	2|0	164|3 - $5\frac{1}{4}$
		£82-3-$5\frac{1}{4}$ *Ans.*

lb.

6*d*	$\frac{1}{2}$	1753 *lb.* at 11$\frac{1}{2}$
4*d*	$\frac{1}{3}$	876 - 6
1$\frac{1}{2}$	$\frac{1}{4}$	584 - 4
		219 - 1$\frac{1}{2}$
	2|0	167|9-11$\frac{1}{2}$
		£83-19-11$\frac{1}{2}$ *Anſ.*

		lb.
6*d*	$\frac{1}{2}$	1753 at 11$\frac{3}{4}$
4*d*	$\frac{1}{3}$	876 - 6
1$\frac{1}{2}$	$\frac{1}{4}$	584 - 4
$\frac{1}{4}$	$\frac{1}{6}$	219 - 1$\frac{1}{2}$
		36 - 6$\frac{3}{4}$
	2|0	171|6-5$\frac{3}{4}$
		£85-16-5$\frac{3}{4}$ *Anſ.*

		lb.
1*s*	2|0	175|3 at 12*d*.
		£87-13 *Anſ.*

And thus, *Tyro*, I have ſhewn you from one farthing to a ſhilling, how to take the parts, ſo as to do any ſum for any number of *lbs. yards*, &c. You may by your own practice take the parts in what manner you pleaſe, only remember this *ſtanding rule*, That when you take the parts of a ſhilling, you muſt divide the top number by ſuch parts; but when you take the parts of any part, you muſt divide ſuch a part by the number of parts, and not the top line. Do you underſtand it?

Tyro. I do, Sir, very well; but pray, ſuppoſe the price exceeds one ſhilling, how muſt I do then?

Philo. That you will ſoon learn by the following CASE.

CASE

CASE 3.

When the given price exceeds one ſhilling, or 12 pence, but is leſs than two ſhillings, then let the top line or given number, ſtand as it is given, without drawing any line under it; for that is the price or value at one ſhilling; then take the parts, as before, out of the top line, or given number, and add the ſaid parts and top number together, you have the anſwer in ſhillings, pence, and farthings, which divide by 20, as before.

EXAMPLES *above twelve pence, and under two ſhillings.*

3*d* | $\frac{1}{4}$ | 1753 *lb.* at 15*d.*
438 - 3*d.*

2|0 | 219|1 - 3

£109-11-3 *Anſ.*

Here you ſee that I let 1753 ſtand without drawing a line, and that is of itſelf an anſwer for 12*d.* a *lb.* Then I ſay, as before, 3*d.* is $\frac{1}{4}$, and I add theſe two together, muſt be the price at 15*d.* for 12 and 3 is 15.

6*d* | $\frac{1}{2}$ | 1753 *ells* at 19*d.* $\frac{1}{2}$
$1\frac{1}{2}$ | $\frac{1}{4}$ | 876 - 6*d.*
219 - $1\frac{1}{2}$

2|0 | 284|8 - $7\frac{1}{2}$

£142-8-$7\frac{1}{2}$ *Anſ.*

EXAMPLES *of other numbers.*

6*d* | $\frac{1}{2}$ | 47 *yards* at 21*d.* $\frac{1}{2}$
3*d* | $\frac{1}{2}$ | 23 - 6*d.*
$\frac{1}{2}$ | $\frac{1}{6}$ | 11 - 9
1 - $11\frac{1}{2}$

2|0 | 8|4 - $2\frac{1}{2}$

£4-4-$2\frac{1}{2}$ *Anſ.*

6*d.*

6d	$\frac{1}{2}$	*85 *ells* at 22*d*. $\frac{1}{2}$
3d	$\frac{1}{2}$	42 - 6
$1\frac{1}{2}$	$\frac{1}{2}$	21 - 3
		10 - $7\frac{1}{2}$
	2|0	15|9 - $4\frac{1}{2}$
		£7-19-$4\frac{1}{2}$ *Anſ.*

* There is a ſort of *mechanical* method of doing ſums, or caſting up the total of the price of any commodity, which is often uſed in buſineſs by ſuch as are not acquainted with taking the parts according to the ſtrict order of this rule.

A mechanical or cuſtomary Way of caſting up any of the foregoing examples for thoſe that cannot divide by the parts.

Let us take the laſt *example, viz.* 85 *ells* at 22*d*. $\frac{1}{2}$ *per* ell.

Rule. Count how many ſhillings, 6 pences, groats, pence, or any other denomination the given number makes, and ſet the products under one another, as in *Addition*, and add all together, you have the anſwer.

Thus, the laſt *example* 85 *ells* at 22*d*. $\frac{1}{2}$ *per* ell.

Firſt,	85 ſhillings	is	£4 - 5 -
	85 ſix-pences		2 - 2 - 6
	85 three-pences		1 - 1 - 3
	85 pence		- 7 - 1
	85 half-pence		- 3 - $6\frac{1}{2}$
			£7 - 19 - $4\frac{1}{2}$ as before

Tyro. I heartily thank you, Sir, for theſe obſervations.

Philo. I do it to ſerve you, take which way you pleaſe; for that is beſt which is ſooneſt underſtood, tho' it be a little deviating from the ſtated rule itſelf. And now,

now, *Tyro*, we will proceed to the other part of this rule.

CASE 4.

Rule 1. When the given price is above two ſhillings, then (by *table* 2) take ſuch ſhillings as even parts of a £. and divide the given quantity thereby; then take the pence as parts of a ſhilling, and the farthings as parts of a penny, dividing each ſum or line by the parts to which they belong. Or,

Rule 2. Multiply the given quantity by the ſhillings, and then take the parts for the pence and farthings, as in the laſt caſes; then add all together, and divide by 20, it is done.

		Yards
2*s.*	$\frac{1}{10}$	125 at 2*s.* 8*d.* $\frac{1}{2}$ *yd.*
6*d*	$\frac{1}{4}$	12 - 10*s.*
2*d*	$\frac{1}{3}$	3 - 2 - 6*d.*
$\frac{1}{2}$	$\frac{1}{4}$	1 - 0 - 10
		- 5 - 2$\frac{1}{2}$
		£16· 18 - 6$\frac{1}{2}$ *Anſ.*

Here 2*s.* is the $\frac{1}{10}$ of a £. therefore I divide 125 by 10, and it gives £12-10*s.* the price at 2*s.* *per* yard. Then 6*d.* is $\frac{1}{4}$th of 2*s.* I therefore divide £12-10*s.* by 4, and it gives £3-2-6. Then 2*d.* is $\frac{1}{3}$ of 6*d.* I therefore divide £3-2-6 by 3, and it gives £1 and 10 pence, the price at 2*d.* *per* yard. *Laſtly*, I ſay 1 half-penny is $\frac{1}{4}$ of 2*d.* and take the $\frac{1}{4}$ of £1 - 10*d.* is 5*s.* 2*d.* $\frac{1}{2}$, the price at a halfpenny a yard. The ſum of all theſe is £16-18-6$\frac{1}{4}$. *Anſ.*

PROOF

PROOF *by the second* RULE.

		Yards
d		125 at 2*s.* 8*d.* $\frac{1}{2}$
6*	$\frac{1}{2}$	2
		250
		62 - 6*d.*
2*d*	$\frac{1}{3}$	20 - 10
$\frac{1}{2}$	$\frac{1}{4}$	5 - 2$\frac{1}{2}$
		33|8 - 6$\frac{1}{2}$
		£16-18-6$\frac{1}{2}$ as before

* Here I multiply 125 by 2, and it gives 250 shillings, at 2*s.* a yard. Then I say 6*d.* is $\frac{1}{2}$ of a shilling, therefore I divide the top line 125 by 2, (and not 250) and it gives 62*s.* 6*d.* and so for any other sum; then I take the other parts, as before directed, as by the work appears.

Philo. Do you understand it?

Tyro. I do very well; but I like the last way the best.

Philo. Take which you please, 'tis only use that makes both easy.

		Ells
5*s*	$\frac{1}{4}$	417 at 5*s.* 10*d.*
d.		
10	$\frac{1}{6}$	104 - 5*s.*
		17 - 7 - 6*d.*
		£121 - 12 - 6 *Ans.*

First, 5*s.* is $\frac{1}{4}$ of a £. I divide 417 by 4, gives £104 - 5*s.* then 10 pence is $\frac{1}{6}$ of 5*s.* therefore I divide £104 - 5*s.* by 6, and find the quotient £17, and 2 over, this 2 I call 2 pounds, and the odd 5*s.* makes £2 - 5, or 45 shillings; then I say the 6's in 45 is 7 times, and 3 over; this I call 3 shillings, and divide that also by 6, it gives 6 pence.

PROOF.

PROOF.

4d	$\frac{1}{3}$	417 *ells* at 5*s*. 10*d*.
		5
4d	$\frac{1}{3}$	2085
		139
2d	$\frac{1}{2}$	139
		69 - 6*d*.
	2|0	243|2 - 6
		£121-12-6 as before

Here I multiply by 5, and it makes ſhillings; then I ſay 4 pence is $\frac{1}{3}$ of a ſhilling, and take it out of the top line 417: I ſay again 4 pence is $\frac{1}{3}$, and ſet down the ſame; then 2 pence is $\frac{1}{2}$ of 4 pence, and having added all up, and divided by 20, I have the ſame anſwer.

And now, *Tyro*, I ſhall ſet you a few examples with words at length.

10*s*	$\frac{1}{2}$	57 pieces at 12*s* 9*d* $\frac{1}{2}$
s. *d*.		
2·6	$\frac{1}{4}$	28 - 10 -
3*d*	$\frac{1}{10}$	7 - 2 - 6
$\frac{1}{2}$	$\frac{1}{6}$	- 14 - 3
		- 2 - 4$\frac{1}{2}$
		£36 - 9 - 1$\frac{1}{2}$ *Anſ*.

PROOF.

6*d*	$\frac{1}{2}$	57 pieces at 12*s* 9*d* $\frac{1}{2}$
		12
		684
3*d*	$\frac{1}{2}$	28 - 6
$\frac{1}{2}$	$\frac{1}{6}$	14 - 3
		2 - 4$\frac{1}{2}$
	2|0	72|9 - 1$\frac{1}{2}$
		£36-9-1$\frac{1}{2}$ as before

10*s*.	$\frac{1}{2}$	185 load at 18*s* 10*d*
4*s*	$\frac{1}{5}$	92 - 10
4*s*	$\frac{1}{5}$	37 -
8*d*	$\frac{1}{6}$	37 -
2*d*	$\frac{1}{4}$	6 - 3 - 4*d*.
		1 - 10 - 10
		£174 - 4 - 2 *Anſ*.

PROOF.

6*d*	$\frac{1}{2}$	185 load at 18*s* 10*d*
		18 multiply
4*d*	$\frac{1}{3}$	3330
		92 - 6
		61 - 8
	2|0	348|4 - 2
		£174-4-2 as above

And

And thus, *Tyro*, you may take which method you please; but in the next *case* you are confined to take the parts according to *table* 2.

CASE 5.

When the given price of any thing is more than 20 shillings, or 1 pound, and not so much as 2 pounds, then let the given quantity stand without drawing a line under it, and that is the price, at 1 pound; then take the parts for the shillings and pence, as in the last *case*, and the work is done.

EXAMPLES.

		Ells
10*s.*	$\frac{1}{2}$	45 at £1 - 12 - 6
2*s* 6*d*	$\frac{1}{4}$	22 - 10
		5 - 12 - 6
		£73 - 2 - 6 *Ans.*

Here I let 45 stand without drawing any line under it, which is the price at 1 pound *per* ell; then I take the parts according to the rule in the last *case*, adding all together have £73 - 2 - 6 *Ans.*

		Tons
10*s.*	$\frac{1}{2}$	75 at £1 - 17 - 11$\frac{1}{2}$
5	$\frac{1}{2}$	37 - 10
2*s* 6*d*	$\frac{1}{2}$	18 - 15
5*d*	$\frac{1}{6}$	9 - 7 - 6
$\frac{1}{2}$	$\frac{1}{10}$	1 - 11 - 3
		- 3 - 1$\frac{1}{2}$
		£142 - 6 - 10$\frac{1}{2}$ *Ans*

Here you see 75 tons at £1 *per* ton is £75, which I let stand without drawing any line under it, and then I take the parts for 17*s.* 11*d.* $\frac{1}{2}$, as you find in the margin.

CASE 6.

When the given price of the commodity is above two pounds, then multiply by the pounds gives the anſwer, for the number of pounds, and the ſhillings and pence and farthings muſt be taken out of the top, or given number, as before directed.

EXAMPLE.

10*s*	$\frac{1}{2}$	37 *loads* at £3 - 16 - 9
		3
		111
5*s*	$\frac{1}{2}$	18 - 10
1*s*	$\frac{1}{5}$	9 - 5
6*d*	$\frac{1}{2}$	1 - 17
3*d*	$\frac{1}{2}$	18 - 6
		9 - 3
		£.141 - 19 - 9 *Anſ.*

Here I multiply 37 by £3, which gives £111, and then I ſay, 10*s*. is $\frac{1}{2}$ of a £. and take it out of the top £37, which is £18 - 10*s*. then I ſay 5 ſhillings is $\frac{1}{2}$ of 10, and take the $\frac{1}{2}$ of £18-10*s* which is £9 - 5*s*. and ſo I go on, taking the parts as the work plainly ſhews.

One EXAMPLE *at large.*

		Hhds.
10*s*	$\frac{1}{2}$	15 at £12-18-0 $\frac{3}{4}$
		12
		180
5*s*	$\frac{1}{2}$	7 - 10
2-6	$\frac{1}{2}$	3 - 15
6*d*	$\frac{1}{5}$	1 - 17 - 6
$\frac{3}{4}$	$\frac{1}{8}$	7 - 6
		11 $\frac{1}{4}$
		£193 - 10 - 11$\frac{1}{4}$ *Anſ.*

Here 15 hhds. at £12 *per* hhd. is £180; then I ſay 10*s*. is $\frac{1}{2}$, and take the $\frac{1}{2}$ of £15, the top-number, which is £7 - 10*s*. and ſo proceed, by taking the reſt of the parts, ſaying, 5 ſhillings is the $\frac{1}{2}$ of 10 ſhillings, and take the $\frac{1}{2}$ of £7 - 10*s*. &c.

And

And now, *Tyro*, I think you have had sufficient inſtructions in this rule, if you carefully obſerve them. However, that I may omit nothing that may be uſeful to you in buſineſs, there are ſome ſhorter methods of caſting up things than the rule itſelf teaches; but this is according to the given price, that is when the given price is juſt two ſhillings, or when it is any even number of ſhillings, as in the two following *caſes*.

CASE 7.

When the given price is any even number of ſhillings, as 2, 4, 8, 14, 16, *&c.* Then multiply the given quantity by half the even number of ſhillings, and cut off the firſt figure in the units place with a daſh of your pen, or pencil; then all the figures to the left-hand are ponuds, and the figure you cut off in the units place being doubled, will be ſhillings, and you have the anſwer required.

EXAMPLES.

48 yards at 4*s.* *per* yard.
2 Here the price is 4 ſhillings *per* yard,
———— therefore I multiply the given quantity
£9|6 48 by half the price, (*viz.* 2) and it is
Anſ. £9-12*s* 96; then I cut off the firſt figure (*viz.* 6) and double it, which is 12 for the ſhillings, and the figure 9 is the pounds; ſo is the anſwer £9-12*s.* Again,

328 ells at 12*s.* *per* ell.
6 Here I multiply by 6, which is $\frac{1}{2}$
———— the given price, and cut off the firſt
£196|8 figure, and double it for the ſhil-
Anſ. £196-16*s.* lings, which is 16, and the figures on the left-hand of the daſh are pounds.

More EXAMPLES.

417 at 14*s*.
7
£291|9 *Ans*. £291-18*s*.

Again, 695 ells at 18*s*.
9
£625|5 *Ans*. 625-10

CASE 8.

When the given price is any even number of shillings, and you would know what quantity of any thing may be bought for any even number of pounds sterling, then only add a cypher to the given pounds, and divide that sum by half the number of the given price, gives the answer in pounds sterling.

EXAMPLES.

How many *lb*. of tea, at 12*s*. *per lb*. may I have for £36. *Ans*. 60 *lb*.

£36 add a cypher is 360

$\frac{1}{2}$ of 12*s*. is 6) 60 *lb*. *Ans*.

How many gallons of rum, at 8*s*. *per* gallon, may I have for £250.

£250 add a cypher is 2500

$\frac{1}{2}$ of 8*s*. is 4) 625 gallons. *Ans*.

Tyro. I like this very well, sir; and it is not only easier, but much shorter than taking the parts: but suppose there be fractions after the given number, how then?

Philo. Take such *fractional* parts out of the given price and add it to the rest of the work, as by the following *case*.

CASE

CASE 9.

When the given quantity, or number, has *fractional* parts after it, then take such parts out of the given price, and place the sum under the rest, and add all together. Or thus, (which may be easier) multiply the price by the *numerator*, (or top-figure) of the fraction, and divide the product by the denominator (or lower-figure) and you have the *fractional* value at once.

10s. | $\frac{1}{2}$ | 15 $\frac{5}{8}$ yards at 10s.
| $\frac{1}{8}$ | 7 - 10
6 - 3 for $\frac{5}{8}$ths
£7 - 16 - 3 *Ans.*

Here I say $\frac{1}{8}$th of 10s. is 1s. 3d. therefore, $\frac{5}{8}$ths is 5 times as much, *viz.* 6s. 3d. Or thus, I multiply 10s. the price, by 5, the numerator, which is 50, and divide by 8, the denominator, I have 6s. 3d. the price at $\frac{5}{8}$ths.

3 $\frac{7}{8}$ yards at 18s.
9*
2 - 14
$\frac{7}{8}$ 15 - 9
£3 - 9 - 9 *Ans.*

* Here the price being an even number of shillings, I multiply by $\frac{1}{2}$ of it, *viz.* 9, (according to *case* 7) and double the first figure, which is £2-14s. Then for $\frac{7}{8}$ I multiply the price, *viz.* 18s. by 7, and divide by 8, and it is 15s. 9d. And thus may many sums in this *case*, as well as in others, be done in a very short and easy manner, by care and observation.

Tyro. I see it plainly, sir; and now, if you please, I would be glad to have a notion of working *weight*, *viz. tons, cwts. qrs. lbs. &c.*

Philo. I am as ready to instruct you.

SECTION II.

Of WEIGHT.

Tyro. WHAT is the nature of working this rule?

Philo. By taking the parts, as before directed; as you will plainly see by and by: but first of all the two following tables ought to be learned perfectly.

TABLE 1.

Even parts of a ton.

C.	qrs.		
10	0	is	$\frac{1}{2}$ a *ton.*
5	0	is	$\frac{1}{4}$
4	0	is	$\frac{1}{5}$
2	2	is	$\frac{1}{8}$
2	0	is	$\frac{1}{10}$
1	1	is	$\frac{1}{16}$
1	0	is	$\frac{1}{20}$

TABLE 2.

Even parts of an Cwt.

qrs.	lbs.		
2	0	is	$\frac{1}{2}$ an *Cwt.*
1	0	is	$\frac{1}{4}$
	16	is	$\frac{1}{7}$
	14	is	$\frac{1}{8}$
	8	is	$\frac{1}{14}$
	7	is	$\frac{1}{16}$

Having got the above *tables* pretty readily by heart, the rule is,

RULE.

Multiply the given tons, or hundreds, by the given price, and if there be any parts of shillings and pence, work with them as in the foregoing examples: this being done, take the parts for the odd weight out of the given price, and place it under the rest, and the work is done.

But this is better understood by an example or two.

What

What comes 7 *C.* 3 *qrs.* to, at £2 - 6*s. per Cwt.*

		2
5*s.*	$\frac{1}{4}$	14
1	$\frac{1}{5}$	1 - 15
2 *qrs.*	$\frac{1}{2}$	- 7
1	$\frac{1}{2}$	1 - 3
		11 - 6
		£17 - 16 - 6

Here I multiply 7 by 2, and it gives £14; then I ſay 5*s.* is $\frac{1}{4}$, and take the $\frac{1}{4}$ of £7, the top number, then I ſay 1*s.* is $\frac{1}{5}$ of 5*s.* and take the $\frac{1}{5}$ of £1-15*s.* This being done, I begin with the odd weight, ſaying, 2 *qrs.* is $\frac{1}{2}$ of an *Cwt.* and I take the $\frac{1}{2}$ of the price £2-6*s*, and it is £1-3*s.* then 1 *qr.* is $\frac{1}{2}$ of 2 *qrs.* I take the $\frac{1}{2}$ of £1-3*s.* which is 11. 6*d.*

What coſt 14 *ton*, 6 *C.* 3 *qrs.* 14 *lb.* at £12-10 *per ton?*

		12	
10*s.*	$\frac{1}{2}$	168	— The value at £12 *per ton.*
5*C.*	$\frac{1}{4}$	7	— Ditto at 10*s. per ton.*
1£.	$\frac{1}{5}$	3 - 2 - 6	— The $\frac{1}{4}$ of £12-10*s.*
2 *qrs.*	$\frac{1}{2}$	12 - 6	— The price of 1*C.*
1 *qr.*	$\frac{1}{2}$	6 - 3	— Ditto of 2 *qrs.*
14*lb.*	$\frac{1}{2}$	3 - 1$\frac{1}{2}$	— Ditto of 1 *qr.*
		1 - 6$\frac{3}{4}$	— Ditto of 14 *lb.*
		£179 - 5 - 11$\frac{1}{4}$	*Anſ.*

I have here, *Tyro*, ſet againſt every line what the value is, and where it is taken out of, that you may be the better grounded in the work: for remember the money is taken out of the given weight, and the odd weight is taken out of the given price, as you were told in the rule.

Tyro. I ſee the nature of it, ſir, plainly.

Philo. Then I ſhall leave you a few queſtions for you to work, or prove at leiſure

QUESTIONS *to exercise the learner.*

1. What cost 125 *lb.* of indigo, at 14*d.* $\frac{3}{4}$ *per lb*? *Ans.* £7 - 13 - 7$\frac{3}{4}$.

2. What is gained in £476, at 4*s.* 10*d.* $\frac{1}{2}$ *per* £. *Ans.* £116 and 6*d.*

3. What is the value of 47 load, at £3 - 15 - 6$\frac{1}{2}$ *per* load? *Ans.* £177 - 10 - 5$\frac{1}{2}$.

4. What cost 9 ton, 15 £. 3 *qrs.* 27 *lb.* of iron, at £14 *per* ton? *Ans.* £137 - 3 - 10$\frac{1}{2}$.

DIALOGUE X.

SECTION I.

SIMPLE INTEREST.

Tyro. SIR, you have given me great satisfaction in the rules of *practice*, and now I should be glad to be informed something concerning *interest*. Pray what is it?

Philo. *Interest* is money that arises from a certain sum lent out at so much *per cent. per annum*; and is a consideration allowed by the borrower to the lender.

Tyro. What do you mean by *per cent. per annum*?

Philo. *Per cent.* signifies 100 pounds, and *per annum* signifies a year, or 12 calendar months.

Tyro. How is the interest found?

Phil. *Interest* may be performed by the *Rule of Three Direct*, (as you may see by *example* 1 following) but in some cases it is done easier and shorter by *Practice* and custom, and that is the reason that *Interest* has a place and title in all books of arithmetic by itself.

Tyro.

Tyro. Please to give an example by the *Rule of Three*, and prove it by the short customary way.

Philo. I will.

EXAMPLE.

What is the interest of £175 - 10s. for a year, at £5 *per cent. per ann.*

```
     £.        £.             £.  s.
If  100 —— be 5 ———— what is 175 - 10
     20                           20
   ————                        ————
   2000                        3510
                                  5
                           ————————
                      2000)17550(8£.
                           16000
                           ——————
                            1550
                              20
                           ——————
                      2000)31000(15 shillings
                           2000
                           ——————
                           11000   Ans. £8 - 15 - 6
                           10000
                           ——————
                            1000
                              12
                           ——————
                      2000)12000(6d.
                           12000
                           ——————
                               0
```

Here you see I am obliged to make my *first* and *second* numbers of one name, so that according as the sum is, it requires a great many figures; but by the other method you spare all this trouble; which pray observe.

Second METHOD. CASE 1.

Multiply the principal, or given sum, by the rate *per cent.* and cut off two figures towards the right-hand, (which is the same, observe, as dividing by 100) and the figures towards the left are pounds interest: This done, multiply the figures cut off to the right-hand by 20, and take in the odd shillings, and cut off two figures, as before, and the figures on the left-hand are shillings: then multiply the remainder by 12, and cut off two figures, and the figures on the left-hand are pence. *Lastly*, multiply by 4, and cut off as before, you have the farthings.

Thus the above example is done, as under.

```
        £.    s.
       175 - 10
              5
     ----------
   £8)77 - 10
      20
     ----------
  s. 15) 50
         12
     ----------
  d6)  00     Ans. £8 - 15 - 6 as above
```

Tyro. Indeed this is very short, to what the first work is.

Phil. Then pray observe the rule; for all the following examples are performed after the same manner; and remember, that cutting off two figures towards the right-hand is the same as dividing by 100.

Quest.

Quest. 2. What is the intereſt of £515 - 10*s.* for 1 year, at £4 *per cent. per annum?*

£515 - 10
4 *per cent.* multiply

£20)62
20

s. 12) 40
12

d. 4) 80
4

Anſ. £20 - 12 - 4$\frac{3}{4}$ $\frac{20}{100}$, or $\frac{2}{10}$th of a
qrs. 3)20 farthing

Queſt. 3. What is the intereſt of £1050 for 1 year, at £3 *per cent per annum?*

£1050
3 *per cent.*

£31)50
20

*s.*10)00 *Anſ.* £31 - 10*s.*

Tyro. You need not give me any more examples of this ſort: but ſuppoſe the money be lent for more than one year, how then?

CASE

CASE 2.

When the given rate *per cent.* is an even part of £100, then divide the principal by that even part, and the quotient is the answer at once.

Take *Example* 1, *viz.* £175 - 10*s.* at £5 *per cent.*

Here £5 being $\frac{1}{20}$ of £100, I only divide £175 - 10*s.* by 20, and it is done.

£.	s.	
2|0)17|5	- 10*	
£8	- 15	- 6 *Ans.*

**Note,* After I divide 175 by 20, I say, the 20th part of 10*s.* is 6 pence.

Quest. 4 What is the interest of £345 - 17 - 6, at £25 *per cent.*

Here £25 being $\frac{1}{4}$ of £100, I divide the principal £345 - 17 - 6 by £4, and it is done.

£.	s.	d.
4)345	- 17	- 6
£86	- 9	- $4\frac{1}{2}$ *Ans.*

CASE

CASE 3.

When the principal is put out for years and months, find the intereſt for the years firſt, and take the months as even parts of a year, according to the rules of *Practice*.

Queſt. 5. What is the Intereſt of £175-10*s*. for 3 years 9 months, at £5 *per cent. per ann.*

£175 - 10		
5 *per cent.*		
£8) 77 - 10		
20	intereſt for 1 year is £8-15-6	
		3
s. 15) 50		
12	6 mths. $\frac{1}{2}$ a year	26 - 6 - 6 for 3 yr.
	3 mths. $\frac{1}{2}$	4 - 7 - 9
d. 6) 00		2 - 3 - 10$\frac{1}{2}$
	Anſ.	£32 - 18 - 1$\frac{1}{2}$

All prac-

A practical QUESTION.

Quest. 6. A Gentleman left his niece by will £558 - 15 shillings, to be paid her when she came to age, with interest, at £4 *per cent.* Now she came to age in 5 years, 9 months, and 3 weeks: what has she got to receive in all, principal and interest?

£558 - 15*s.*
4 *per cent.*

£22)35	Interest for 1 year	£22 - 7*s.*
20		5 years
*s.*7)00	6 mths ½ year	111 - 15
	3 mths ½	11 - 3 - 6*d.*
	2 weeks $\frac{1}{6}$	5 - 11 - 9
	1 week ½	- 18 - 7½
		- 9 - 3¾
		£129 - 18 - 2¼ interest
		558 - 15 legacy
	The legacy and interest is	£688 - 13 - 2¼

Tyro. Sir, I thank you for these two useful examples: and now I would only ask this one favour, and that is, how am I to tell the interest of any sum for any time, at 3½, 4½, 4¾, or the like *per cent.*

Philo. It is something more difficult than the other, but an example or two, with your due observation, will make it very easy.

CASE

CASE 4.

When the given rate *per cent.* is not even pounds, but is $2\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{2}$, or the like, then multiply the principal, or given sum, by the even pounds, and take the $\frac{1}{2}$, or $\frac{1}{4}$, or $\frac{3}{4}$ of the said principal, or given sum, and add to that product; then cut off two figures, and proceed in all respects as before.

Quest. 7. What comes the interest of a bond of £685 to, at £$4\frac{1}{2}$ (that is, £4 - 10*s.*) *per cent. per annum?*

s.		685
		4
10	$\frac{1}{2}$	2740
		342 - 10
		£ 30)82 - 10
		20
		s: 16)50
		12
		d. 6)00

Here I multiply by 4, then for $\frac{1}{2}$ *per cent.* which is 10 shillings, I say 10*s.* is $\frac{1}{2}$ as in *Practice*, and take $\frac{1}{2}$ of 685, and add to the other, it is £3082 - 10*s.* then I cut off two figures, thus, 30|82 and proceed as before, I find it £30 - 16 - 6.

Quest.

Queſt. 8. Lent £500 upon a mortgage, to receive intereſt at £4¾ (*viz.* £4 - 15*s.*) *per cent.* till it was paid off: now the mortgage was paid off 3 years, 8 months after: I demand the intereſt due to me?

		£.				
		500	Intereſt for 1 year is			£23 - 15 -
		4	multiply by			3
s.						
10	½	2000	6 months		½	71 - 5 - for 3 yrs
5	¼	250	2 months		⅓	11 - 17 - 6 6 mths
		125				3 - 19 - 2 2 mths
		£23)75				£87 - 1 - 8 *Anſ.*
		20				
		s. 15)00				

Tyro. I underſtand all theſe examples very well; but ſuppoſe the intereſt be for leſs than a year, how then am I to find it?

Philo. Very eaſily.

CASE 5.

When the intereſt of any ſum is to be found for leſs than a year, *viz.* for 1, 2, 3, 4, or 10 months, or for months and odd days, then firſt find the intereſt for 1 year, and take the parts of that year's intereſt according to the given time: thus, if it be 6 months intereſt, take the half of the year's intereſt; if 9 months, take ¾; if 3 months, take ¼; and for the odd weeks, or days, take the parts of thoſe months.

Queſt.

Quest. 9. What is the interest of £175 - 10*s.* for 8 months, at £5 *per cent. per annum*?

£.				
175 - 10	Interest for 1 year is		£8 - 15 - 6	
5				
£8)77 - 10	Then say, 6 mths	$\frac{1}{2}$	4 - 7 - 9	for 6 m
20	2 mths	$\frac{1}{3}$	1 - 9 - 3	for 2 m
			£5 - 17 -	*Ans.*
*s.*15)50				
12				
*d.*6)00				

Here you see I take the half of 1 year's interest, which is £4-7-9 for 6 months; then I say, 2 months is $\frac{1}{3}$ of 6 months, and take the $\frac{1}{3}$ of £4-7-9, and add both together, is £5-17 for 8 months.

Tyro. I am satisfied, Sir.

Philo. This is the best way for calculating interest in general, and will be near enough for common practice, should there be weeks or days; for it is only taking the parts for such weeks and days, as was said before.

Tyro. But suppose the interest be put out for such a number of days as cannot well be taken in their even parts, how then?

RULE I.

Philo. First, find the interest for one year as before; then say, If 365 days give so much, what will so many days give? Thus, suppose the interest for one year be £17-10-10, and it was required to find the interest for 73 days, I say,

If 365 days give £17 - 10 - 10, what will 73 days give? *Ans.* £3-10-2.

RULE

RULE 2.

Bring the principal, or money lent out, into pence, and multiply those pence by the number of days the sum is put out at, or continues out; then if the rate *per cent.* it be put out at be £5, divide by 7300; and if it be £6 *per cent.* divide by 6083, you have the answer in pence. There are many other ways to find the interest for days; but I would recommend the first rule of these two to the learner, as it serves for all rates *per cent.* and is most certain.

SECTION II.

Of Assurance, Brokerage, *or* Commission.

Tyro. WHAT is *Assurance, Brokerage, or Commission?*

Philo. *Assurances*, or *Insurances*, are of various sorts. An *Assurance* is when any person agrees with another at a certain rate *per cent.* to insure his life for such a time, or his ship upon a voyage, from the dangers of the seas, or his house or goods from fire. *Brokerage* is an allowance of so much *per cent.* given to *Brokers*, or persons employed in buying and selling stocks, or transacting business between buyer and seller: And *commission* is also an allowance of so much *per cent.* for buying or selling any sort of commodity, by the order of any other person, *&c.*

Tyro. Then I perceive these are all cast up the same as the interest of money is.

Philo. They are so, only interest is so much *per cent. per annum*; but *brokerage* is cast up only at so much *per cent.* ready money, without any regard to time. Of these in their order.

1. Of

1. *Of* ASSURANCES.

Quest. 1. What comes an assurance of £580 to, at £10 $\frac{3}{4}$ *per cent.*

		£580
		10
s.		
10	$\frac{1}{2}$	5800
5	$\frac{1}{2}$	290
		145
		£62)35
		20
		s.7)00

Ans. 62 - 7

Here I multiply by 10, then I take the parts out of the top, or £580, and add all together, and cut off two figures, (which is dividing by £100) and proceed as in interest, by multiplying the remainder by 20 &c.

Quest. 2. Shipped for *Jamaica* goods to the value of £1200, upon which I made an assurance, at £7 $\frac{5}{8}$ths *per cent.* what does it come to?

		£.
		1200
		7
$\frac{4}{8}$ is	$\frac{1}{2}$	8400
$\frac{1}{8}$ is	$\frac{1}{4}$	600
		150
		£91)50
		20
		s.10)00

Here I multiply by 7. Then I say for the fraction 4 eighths is $\frac{1}{2}$, that is, $\frac{1}{2}$ of $\frac{8}{8}$th (for 8 eighths is equal to 1 whole integer) and take the $\frac{1}{2}$ of the top number; then I say $\frac{1}{8}$th is $\frac{1}{4}$ of $\frac{4}{8}$ths; then I add all together, and cut off two figures, as before, &c. and find the answer £91 - 10*s.*

2. *Of* Brokerage *and* Commission.

Tyro. You need not give me any more examples; for I see the work is the same, tho' under different titles, or names.

Philo. It is so; but still when the *per cent.* is shillings it may be a little difficult to you, therefore, I will give you an example or two.

Quest. 3. What is the commission of a *Broker* for buying or selling £520 stock, at half a crown *per cent.*

Rule. Multiply the money by the shillings, and take the parts for the pence as in *Practice*; then add all together, and cut off two figures to the right-hand, and those towards the left are your answer in shillings.

		£ 520
		2
6*d*	½	1040
		260
		s. 13)00

Ans. 13 shillings.

Quest. 4. What is my commission on £1500 at 7*s.* *per cent.*

	7
shill.	105)00

Answer £5 - 5*s.*

Another way.

Cut off two figures of the given sum, and multiply the remainder by 20, 12, and 4, and gives the answer, at £1 *per cent.* then take the parts of the sum of £1 *per cent.* as in *Practice*, you have your answer.

Quest.

Quest. 5. What is my commission on £1252 - 10*s.* at 7*s.* 6*d.* *per cent.*

£12)52 - 10*s.*	At £1 *per cent.* it is		£12 - 10 - 6
20			
s. 10)50	Then, 5*s.*	$\frac{1}{4}$	3 - 2 - 7$\frac{1}{2}$
12	2-6	$\frac{1}{2}$	1 - 11 - 3$\frac{3}{4}$
d. 6)00		*Ans.*	£4 - 13 - 11$\frac{1}{4}$

3. *Compound* Interest.

What is *Compound Interest?*

Philo. *Compound Interest* is called interest upon interest; that is, the interest of the principal, and the interest of the interest added together, is the *Compound Interest.* But it being seldom used without it be in purchasing of annuities, &c. and being very tedious to calculate, is done with more ease by *Decimal Fractions.*

Tyro. But I should be glad you would give me a little better notion of it.

Philo. Suppose I put out £100 for 3 years, at £5 *per cent. compound* interest, what does it come to?

First, I find the simple interest for one year is £5; then the second year I am to count what the interest of £105 is, and find it £5 - 5*s.* This I add to the last principal £105, and it makes £110 - 5*s.* Then I see what is the interest of £110 - 5*s.* and find it £5 - 10 - 3 for the third year. Then I add these different interests together, *viz.* £5, £5 - 5*s.* and £5 - 10 - 3, and it makes £15 - 15 - 3, the *compound interest* for three years on £100, at £5 *per cent.* And thus for any number of years, at any rate *per cent.*

SEC-

SECTION III.

Of REBATE, or DISCOUNT.

Tyro. WHAT do you mean by *Rebate* or *Discount?*

Philo. Discount is when a sum of money, which is to be paid at any time to come, is satisfied by paying of it down in present money, which present money being put out after the same rate *per cent.* and for the same time, will be increased to the debt or sum that was first to be paid.

Tyro. Explain this a little fuller in other words.

Philo. Suppose a person owes me 100 guineas, *viz.* £105 to be paid in a year, or 12 months to come, how much present money will satisfy the debt? *Ans.* £100 present money will satisfy a debt of £105 due 12 months hence; because £100 put out to interest 12 months will be £5.

There are several methods of performing this rule of *Discount*; but as it is best done by *Decimal Fractions*, I shall only give this one general rule.

A general RULE.

1. Add the interest of £100 for the given time to £100, and make this your *first number*; then place £100 only for your *second* number, and the given debt your *third number*, and the answer will be the present money that will satisfy the debt due 12 months to come; then take this present money from the debt, and the remainder is the *rebate* for 12 months.

2. Having thus found the *rebate* for one year, if the time be more or less than 12 months, take the parts by the rule of *Practice* accordingly.

3. Thus

3. Thus it appears that the *firſt number* is always known according to the rate *per cent* for one year, as by the following table.

For at £3 *per cent.* the *firſt number* will be 103, &c.

£ 3 *per cent.*	103	These obſerve are *firſt* numbers according to the *Rebate per cent.* for one year, £100, is your *ſecond*, and the debt your *third*, and the anſwer is the preſent money to be paid down.
4	104	
5	105	
6	106	
7	107	
8	108	

Queſt. 1. What preſent money will pay a draught or note of hand of £368 - 4*s.* due 3 months hence, allowing rebate at £5 *per cent. per annum?*

If 105 —— 100 ——368 - 4. *Anſ.* 350-13-4, which taken out of the debt £368 - 4*s.* leaves 17 - 10 - 8 the diſcount for one year. See the work.

Rule. Bring the *firſt* and *third* number into ſhillings; then multiply the *third* by your *ſecond*, and divide by your *firſt*, you have £ 350, and 14 remains, which alſo multiply by 20, and divide by your *firſt number*, you have 13 ſhillings, and 7 remains, which multiply by 12, and dividing by the ſame *firſt* number you have 4*d.*

Note of hand		£ 368 - 4 -	
Preſent money		350 - 13 - 4	
Diſcount is		17 - 10 - 8	for 1 year
3 months	$\frac{1}{4}$	4 - 7 - 8	*Anſ.* for 3 months.

Which taken out of £368-4*s.* remains £363-13-4 preſent money to be paid down.

Here

Here you see for one year the *discount* is £17 - 10 - 8, and as 3 months is ¼th of a year, I take the ¼th of that sum, which is £4 - 7 - 8 *Ans.*

Note, Had the note been drawn for 15 months, that is, 1 year and 3 months; then add £4 - 17 - 8, the *discount* of 3 months to £17 - 10 - 8, which is £21 - 18 - 4, the *discount* of £368 - 4*s*. for 15 months.

An EXAMPLE *for practice.*

What present money will pay a debt of £500, due 3 years, 4 months hence, allowing *rebate*, at £6 *per cent. per annum? Ans.* £416 - 13 - 4. *Rebate* £83 - 6 - 8. For by the table your *first* number will be £106 for one year, your *second* £100, and the *third* the debt. And, for proof, find the interest of £416 - 13 - 4 for three years and four months, at £6 *per cent.* and add to it £416 - 13 - 4, you will have £500 the debt itself.

And by these examples you may find the *discount* at any rate *per cent.* and for any time, by observing the foregoing directions.

Tyro. I see the manner of working this rule very plainly.

Philo. Then I will give you a notion of another useful rule, *viz.*

SECTION IV.

EQUATION *of* PAYMENTS.

Tyro. WHAT do you mean by *Equation of Payments?*

Philo. Equation of Payments is when a certain sum of money becomes due at different times, or is to be paid

paid at different payments, and it is agreed between the debtor and creditor to set, or pitch upon a certain time for the payment of the whole, without any damage or loss to either.

Tyro. This is very pretty, as well as useful: pray give me the rule for working questions of this sort.

RULE.

Multiply the sum of each particular payment by the time it is to be paid in; then add the several products together, and divide the sum by the whole debt, and the quotient is the *equated*, or just time for the payment of the whole.

Quest. 1. A merchant buys goods to the value of £140, £30 of which is to be paid in 1 month, £40 in 3 months, £20 in 4 months, and £50 in 6 months; but it is agreed upon an *equated* time to pay the whole at once; I demand the *equated*, or just time of payments?

I set down each payment, and the time right against it, and multiply the money by the time, and set the products of each right against them; as follows.

£ 30 multiplied	by 1 mth	is	£ 30
40	by 3 mths	is	120 Add
20	by 4 mths	is	80
50	by 6 mths	is	300

Debt 140)530(3 months
420
———
110
30 * *
———
140)3300(23 days
280
———
500
420
———
80

Ans. 3 months, 23 days

* * *Note,*

Here you ſee for one year the *diſcount* is £17 - 10 - 8, and as 3 months is ¼th of a year, I take the ¼th of that ſum, which is £4 - 7 - 8 *Anſ.*

Note, Had the note been drawn for 15 months, that is, 1 year and 3 months; then add £4 - 17 - 8, the *diſcount* of 3 months to £17 - 10 - 8, which is £21 - 18 - 4, the *diſcount* of £368 - 4*s*. for 15 months.

An EXAMPLE *for practice.*

What preſent money will pay a debt of £500, due 3 years, 4 months hence, allowing *rebate*, at £6 *per cent. per annum?* *Anſ.* £416 - 13 - 4. *Rebate* £83 - 6 - 8. For by the table your *firſt* number will be £106 for one year, your *ſecond* £100, and the *third* the debt. And, for proof, find the intereſt of £416 - 13 - 4 for three years and four months, at £6 *per cent.* and add to it £416 - 13 - 4, you will have £500 the debt itſelf.

And by theſe examples you may find the *diſcount* at any rate *per cent.* and for any time, by obſerving the foregoing directions.

Tyro. I ſee the manner of working this rule very plainly.

Philo. Then I will give you a notion of another uſeful rule, *viz.*

SECTION IV.

EQUATION *of* PAYMENTS.

Tyro. WHAT do you mean by *Equation of Payments?*

Philo. *Equation of Payments* is when a certain ſum of money becomes due at different times, or is to be paid

paid at different payments, and it is agreed between the debtor and creditor to ſet, or pitch upon a certain time for the payment of the whole, without any damage or loſs to either.

Tyro. This is very pretty, as well as uſeful: pray give me the rule for working queſtions of this ſort.

RULE.

Multiply the ſum of each particular payment by the time it is to be paid in; then add the ſeveral products together, and divide the ſum by the whole debt, and the quotient is the *equated*, or juſt time for the payment of the whole.

Queſt. 1. A merchant buys goods to the value of £140, £30 of which is to be paid in 1 month, £40 in 3 months, £20 in 4 months, and £50 in 6 months; but it is agreed upon an *equated* time to pay the whole at once; I demand the *equated*, or juſt time of payments?

I ſet down each payment, and the time right againſt it, and multiply the money by the time, and ſet the products of each right againſt them, as follows.

£ 30 multiplied	by 1 mth	is	£ 30
40	by 3 mths	is	120 add
20	by 4 mths	is	80
50	by 6 mths	is	300

Debt 140)530(3 months
420
———
110
30 * *
———
140)3300(23 days
280
———
500
420
———
80

Anſ. 3 months, 23 days

** *Note*, I multiply by 30, the days in a calendar month, according to the cuſtom of buſineſs, becauſe 12 ſuch months are reckoned to the year.

Queſt. 2. A merchant bought goods to the value of £500 upon the following conditions, *viz.* ¼th to be paid in 2 months, and the remainder in 6 months; I demand the *equated* time to pay the whole at one payment?

Firſt, ¼ of £500 is	£125	multiplied by 2 mths	£250
The remainder is	375	multiplied by 6 mths	2230
Debt	£500		5|00)25|00
			Anſ. 5 mths

Now £2500 divided by the debt £500 is 5 months for the anſwer, and thus for any other queſtion. Do you underſtand it?

Tyro. There can be nothing eaſier to apprehend than this.

QUESTIONS *to exerciſe this* RULE.

3. A father left his ſon £600, to be paid by his executor, as follows, ⅓ in 2 months, ⅓ in 3 months, and ⅓ in 8 months; but he being very troubleſome to the executor, and he as willing to be rid of him, they agree upon a time for the payment of the whole at once. The *equated* time is demanded; *Anſ.* 4″3 months, *viz.* 4 months 10 days.

4. *A* owes *B* £2000 to be paid 3 months hence; but *A* agrees to pay him £1200 down, provided he will give him a longer time for the payment of the remainder. The queſtion is, how long time *B* ought to give him to pay the other £800 in? *Anſ.* 7½ months.

Tyro.

Tyro. What other rules are there in common *arithmetic?*

Philo. Several others, as *Alligation Medial*, and *Alternate*, *Single* and *Double Position*, *Composition* of *Medicines*, *Arithmetical* and *Geometrical Progression*, &c. but these being not so much required in business, (or at least may be done by the *Rule of Three Direct*) I shall give a little notion of the two last, because it may be of service to you in case your fancy should turn to any part of the *mathematical* studies.

DIALOGUE XI.

SECTION I.

Of ARITHMETICAL PROGRESSION.

Tyro. WHAT do you mean by *Arithmetical Progression?*

Philo. *Arithmetical Progression* is by some called *Comparative Arithmetic*, because in a series of numbers one is compared with another, in order to know what ratio or proportion one bears to the other; that is, in what manner they differ, and how much, and that difference is always equal in numbers that are continued.

OBSERVATION.

1. Let us take any numbers that differ alike from one another: thus, 3, 6, 9, 12, 15, &c. differ by 3, called the common excess, difference, or *ratio*. Also, 1, 9, 17, 25, 33, are numbers in *Arithmetical Progression*, whose *ratio*, or common difference is 8.

 2. In

2. In any series of figures in *Arithmetical Progression*, when the number of places is odd, *viz.* 3, 7, 11, 15, 19, or the like, the double of the middle figure, or place, is equal always to the sums of the first and last figure; thus, 2, 6, (10) 14, 18. The double of the middle term 10 is equal to the sum of the first and last number, *viz.* 20. So 3, 8, 13, (18) 23, 28, 33. The middle term 18 doubled is equal for the sum of 3, and 33, *viz.* 36.

Tyro. How many cases are there in this *rule?*

Philo. Six; but *two* of them will be sufficient for your purpose in working questions in general, or to give you a true notion of the nature of the rule itself.

CASE 1.

The number of places, or terms, and the *ratio*, or common excess, being given to find that last number,

Multiply the number of places less by 1, by the *ratio*, or common excess, and to that product add the first number, and that sum will be equal to the last number.

1. Let the first number be 3, number of places 7, and common excess 5, what is the last number?

Here the number of places is 7, therefore, I count them for 6, (which is less 1) and the *ratio*, or common excess, is 5; now 5 times 6 is 30; then I add the first number, *viz.* 3, and it is 38, the last number; and so on if the places were ever so many.

2. Let the number of terms be 121, the first number 5, and the common excess, or *ratio*, 9; I demand the last number?

Here

Here the number of places less by 1 is 120, which multiplied by 9, the *ratio* gives 1080, to which add the first number 5, gives 1085, the last number. And this you may prove by setting them down thus, 5, 14, 23, 32, 41, till you have 121 places, and your last number will be 1085. But how tedious is this! since you see the last number is so easily found.

A QUESTION *for Exercise.*

A traveller sets out, and the first day went 6 miles, the second day 9, increasing every day's journey 3 miles, and travelled 61 days; how many miles did he go the last day? *Ans.* 186 miles. See the next *Case.*

CASE 2.

The extremes, that is, the first and last number, and the number of terms being given to find the aggregate, or total sum of all the series of numbers,

Add the first and last number together, and multiply that sum by half the number of places, and the product is the sum of all the series. Or, in case the number of terms be odd, then add the first and last numbers together, and multiply that sum by the number of the terms, and divide that product by 2, and you have the total of all the places.

Thus, in the last question concerning the travels, the first number is 6, the last number 186 miles, and the places 61. Therefore, by the rule, add the first number 6, and the last number 186 together, and it is 192, which I multiply by 61, and it is 11712, and dividing by 2, I have 5856, the miles he went in all in 61 days.

A QUESTION *for Exercise.*

Quest. 2. Suppose 100 stones be placed in a straight line, at a yard distant from each other, and a person undertakes to pick up one at a time, and bring it back to the place where he first set out; how far will he have gone when he has picked up the whole?

Here the number of places are 100, and as the stones are but one yard from each other, the common excess is but 1, and therefore the last number is also 100. But here you are to consider, the man is to come back with every stone; therefore, when he fetches the first stone in coming back, he goes 2 yards to put in the first stone, therefore 2 is your first number, and by the same rule 200 will be your last number.

Last number	200
First number	2
	202
Half of the number of places	50

He goes in all 10100 yards, which reduced into miles, is $5\frac{1}{2}$ miles, and 420 yards, which wants but 20 yards of $5\frac{3}{4}$ miles.

Quest. 3. Suppose stones were laid one yard asunder, in a right line, for one mile in length, and to be picked up one by one, coming back with one at a time; how far would the person go that performs it? *Ans.* 1761 miles.

SEC-

SECTION II.

Of GEOMETRICAL PROGRESSION.

Tyro. I Am mightily pleaſed with what you have ſhewn me; but pray what is *Geometrical Progreſſion*, and wherein does it differ from the other?

Philo. *Geometrical Progreſſion*, or *Proportion*, is when numbers differ from one another, by the like *ratio*, or reaſon, as in *Arithmetical Progreſſion*, only with this difference, that the former is the effect of *Addition*, and here of *Multiplication*, all the numbers having one common multiplier. As 2, 4, 8, 16, differ by double reaſon, or the *ratio* is 2; for twice 2 is 4, twice 4 is 8, *&c* So alſo, 1, 7, 49, 343, *&c.* Here the *ratio* is 7, for 7 times 1 is 7, and 7 times 7 is 49, *&c.* So that you ſee it is *Multiplication.*

Tyro. I thank you Sir: What elſe pray?

NOTE 1.

Philo. Any three numbers differ in *Geometrical Progreſſion*; the product of the *extremes*, that is, the product of the *firſt* and *laſt* number, is equal to the ſquare of the *mean*, that is, equal to the middle number multiplied by itſelf.

Let 4, 12, and 36 be the 3 numbers, whoſe *ratio* is 3, that is, each is 3 times more than the foregoing number; then will the product of the extremes 4 and 36 be equal to the ſquare of 12, the *mean*, *viz.* equal to 144. So alſo 2, 16, and 18. Here 128 multiplied by 2 is equal to 16 multiplied by 16 equal to 256.

NOTE 2.

Any 4 numbers that differ in *Geometrical Proportion*, either continued or interrupted (provided the interruption or breaking off be between the *ſecond* and *third*

 number)

number) the product of the *means* is equal to the product of the *extremes*; that is, the product of the *first* and *fourth* is equal to the product of the *second* and *third* numbers.

Let the 4 numbers be 3, 12, 48, and 192, whose *ratio* is 4. Here 192 multiplied by 3, the *first* number, is equal to 48 by 12, *viz.* equal to 576.

Again, Suppose the numbers be interrupted, as 5, 30, 448, and 2688. Here the *second* term is 6 times more than the *first* term, and the *fourth* term 6 times more than the *third*: therefore, 2688 multiplied by 5 is equal to 448 multiplied by 30.

Note, This mark (∻) stands for *Geometrical Progression*.

NOTE 3.

The *ratio* of any numbers in ∻ is found by dividing any consequent by its antecedent; that is, dividing any number by the foregoing number. Thus, the *ratio* of the last numbers is found by dividing 30 by 5, or 2688 by 448, which gives 6, the *ratio*.

Tyro. I understand this very well; but in a long series of numbers, how am I to find the last number?

Philo. This is very easily done, and as you have not the rule in many, or, at least, have it but for certain *ratio*, or difference of numbers, I shall give you one standing rule, be the *first* number and the *ratio* what it will.

CASE 1.

To find the last number in any series in ∻ having the *first* number and *ratio* given, set down your *first* number, and multiply it by the *ratio*, and that product again by the *ratio*, or common difference, and thus go on for 5 or 6 terms, at pleasure; then multiply any of

of those places by itself, and divide the product thereof by the *first* number, and it will be the double of that number, wanting one place. Thus, suppose you multiply the 6th place by itself, and divide by the *first* number, the quotient will be the 11th place; then, if you multiply the 11th place by itself, and divide by the *first* number, the quotient will be the 21st place, which is the double of 11, wanting 1.

☞ N. B. When the first term is unity, or 1, there is no occasion to divide; for having multiplied by the *ratio* a few times, as before directed (suppose as far as the 6th place) this term multiplied by itself will be the product of the double of terms wanting 1, as was said before.

Tyro. I am obliged to you, Sir; but how shall I find the sum of all the series?

Philo. By the next case.

CASE 2.

The ratio *(or common difference) and the last number being given to find the aggregate, or total sum of all the series,*

1. Multiply the last term, or number, by the common difference, or *ratio*, and from that product take the *first* number; then divide the remainder by the common difference, less 1, and the quotient will be the sum of all the series. Or,

2. Multiply the *second* and last term together; then multiply the *first* term into itself, and take that product from the product of the *second* and last term; then divide the remainder the *second* term, less 1, the *first* term will give the sum of all the series.

Tyro. Please to give me one example.

Philo. I will give you a practical question to exercise both cases.

Quest. A cunning Jockey had a fine gelding, to which a Gentleman took a particular fancy, and after many words had paſſed between them, the Jockey agreed to ſell him to the Gentleman for the price his ſhoes would come to at one farthing for the *firſt* nail, and to double the price every nail. Now the number of nails in the gelding's ſhoes were 28, I demand then what he was ſold for at this rate?

Now, obſerve, *Tyro*, here the common exceſs is 2, (that is, a farthing a nail doubled) Now to find the laſt number by *Caſe* 1, double a few places, ſuppoſe as far as the 7th, which will be 64; then multiply 64 by itſelf, *viz.* 64, you have 4096, the 13th place, which doubled, gives the 14th place, *viz.* 8192. This multiplied by itſelf gives the 27th place, which doubled, gives the 28th, or laſt place, *viz.* 134217728 farthings; then proceed according to *Caſe* 2, you will find the ſum of all the ſeries to be 268435455 farthings, which is £279620 5*s.* 3*d.* ¾. a ſum too large for the ignorant purchaſer. And had there been but 4 nails more in his ſhoes (*viz.* 32) he would have come to £4473924 5*s.**

* It is no eaſy taſk to make ſome perſons believe the truth of the increaſe of figures in Geometrical Progreſſion in theſe queſtions; but ſuppoſe a ſervant was to agree with his maſter to ſerve him thirty years, for 1 ſingle wheat corn the firſt year, for the ſecond year ten, for the third 20, and ſo on. The produce of the wheat would be more than all the ſhips in England could carrry away at once; and the money for his wages would be more than all the land could pay, if ſold at 20 years purchaſe for ever.

SEC-

SECTION III.

Of PERMUTATION, *or variety of changes.*

Tyro. I Imagine this is much after the same manner as *Progression*; is it not?

Philo. This shews the increase of numbers as well as the other; but does not teach you to find the sum of all, only their different variations or changes, as follows.

Multiply every number together, that is, the *first* by the *second*, and that product by the *third* figure, and so on till you have gone through all the given numbers; so is the last product the variety of changes.

EXAMPLE.

I demand the changes that may be rung on 12 bells, or the different position 12 persons may sit at table. *Ans.* 479001600.

☞ *Note,* This sign (+) signifies, that all numbers between which it stands are to be multiplied continually one into the other, thus: take the 12 bells, and multiply the *first* by the *second*, you have two changes upon two bells; multiply this by the *third* bell, you have 6 on three bells; multiply this product by 4, or the *fourth* bell, you have 24 changes, &c. as follow.

Thus, $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12$ will produce 479001600 changes, or different positions which you may prove very easily at leisure by *Multiplication* only*.

* Notwithstanding the changes on 8 bells may be rung out in a few hours; yet the changes on 12 (allowing 10 changes in a minute) will take 91 years, 26 days; and to add to the incredibility of this increase, two bells more, viz. 14, would take up 18575 years to ring the changes through. See Mr. Ward, p. 85.

Tyro.

Tyro. I thank you for giving me a ſhort hint of theſe things, which I perceive the nature of very plainly: And now, pray, what will you inſtruct me in next?

Philo. There are two or three rules, ſuch as the *Rule of Falſe*, *Allegation*, *Alternate*, *Compoſition of Medicines*, which are not material in buſineſs, and therefore for want of room muſt be omitted, and the more neceſſary part, *viz.* *Vulgar Fractions*, be treated of in their ſtead. And here I beg you would take care and make yourſelf maſter of *Vulgar Fractions*, they being very neceſſary in almoſt every branch of life, and the very foundation of *Decimals*.

DIALOGUE XII.

SECTION I.

Of Notation of VULGAR FRACTIONS.

Tyro. WHAT do you mean by *Notation*?

Philo. *Notation*, like *Numeration*, ſhews you how to note, write down, or expreſs any *Fraction*.

Tyro. What is a *Vulgar Fraction*?

Philo. It is a broken number, or a part or parts of an integer, or whole number, and conſiſts of two parts, *viz.* the *Numerator*, which always ſtands a-top, and the *Denominator*, which always ſtands under it, or below. Thus, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{11}{12}$. Here 4, 3, and 11 are the *Numerators*, and 5, 4, and 12 the *Denominators*.

Tyro. How many ſorts of *Vulgar Fractions* are there?

Philo. Three, *viz.* *Firſt*, *Simple*, *Single*, or *Proper Fractions* (for theſe are all one.) *Secondly*, *Improper*; and *thirdly*, *Compound Fractions*.

Tyro. What is a *Simple Fraction*, or how is it known?

Philo.

Philo. *Simple Fractions*, have their *Numerators* leſs than their reſpective *Denominators*. Thus, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{14}{87}$, are all *Simple Fractions*.

Tyro. What is an *Improper Fraction?*

Philo. *Improper Fractions* have their *Numerators* larger than their *Denominators*. Thus, $\frac{4}{3}$, $\frac{9}{5}$, $\frac{87}{14}$, $\frac{127}{2}$, &c. are *Inproper Fractions*.

Tyro. What are *Compound Fractions?*

Philo. *Compound Fractions* are *Fractions of Fractions*, compounded, or joined together by the word *of*. Thus, $\frac{2}{3}$ of $\frac{3}{5}$, or $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{12}$ are *Compound Fractions*, and are thus read, 2 thirds of 3 fifths, or 3 fourths of 5 ſixths of 11 twelfths of an *integer* or *whole number*.

Tyro. I underſtand you, Sir; but this is very difficult to tell ſurely.

Philo. You have nothing to do with this at preſent; you will know it by and by.

Tyro. What do you mean by a *Mixt Number?*

Philo. A *Mixt Number* is a *whole* number with a Fraction after it. Thus, 14 $\frac{2}{3}$, and 147 $\frac{5}{8}$ are *Mixt Numbers*.

Tyro. I underſtand you; but how am I to find the value of different *Fractions*.

Philo. By *Reduction*, which is always learned firſt, becauſe you can't *add*, *ſubtract*, *multiply* or *divide*, till the fractions are firſt reduced to their proper order: but before you proceed any farther, it will be proper to learn the following ſigns, or characters, by heart, ſo as to know what they ſignify, which will be a great help to you.

Of the Signs uſed in Vulgar Fractions.

This (=) is the ſign of Equality, and ſignifies, that the numbers before it are equal to thoſe after it. Thus, 5 and 2=7 is thus read, 5 and 2 is *equal to* 7.

This mark (+) is the ſign of addition, and ſignifies, all the numbers are to be added together, Thus, 2+6+7=15: That is, 2 *more* 6 *more more* 7, or 2 added to 6 and 7 is *equal* to 15.

This

This (—) is the sign of *subtraction*, and signifies the number after it is taken, or is to be taken out of the number before it. Thus, $15-7=8$, that 7 taken, or subtracted from 15, is *equal to* 8.

This (×) is the sign of continual multiplication.

This (÷) is the sign of *division*, and signifies the number before it is to be divided by the number that follows it. Thus, $56\div 8=7$, that is, 56 divided by 8 is *equal to* 7.

SECTION II.

Reduction of Vulgar Fractions.

CASE I.

To reduce a mixt number *to an* improper fraction.

MULTIPLY the whole number by the denominator of the fraction, and take in the numerator besides; then place the denominator under the product, and it is done.

Reduce $57\frac{3}{5}$ to an *improper fraction*?

$$\begin{array}{r} 57\frac{3}{5} \\ 5 \\ \hline 288 \\ \hline 5 \end{array} \quad \textit{Anſ.}\ \frac{288}{5}$$

Reduce $1753\frac{11}{12}$ to an *improper fraction*. *Anſ.* $\frac{21047}{12}$.

CASE

CASE 2.

To reduce an improper fraction *to a* whole *or* mixt number.

Divide the *numerator* by the *denominator*, and if any thing remains, place it for a new *numerator* over the *denominator*.

Reduce $\frac{288}{5}$ to a *mixt number?*

$$5)288$$

$$57\tfrac{3}{5} \text{ Ans.}$$

Reduce $\frac{21047}{12}$ to a *mixt number? Ans.* $1753\frac{11}{12}$.

☞ This is only a proof to the foregoing *case*, and is the same question backwards.

CASE 3.

To reduce or make a whole number into an improper fraction.

Multiply the whole number by any figure at pleasure, and place the product over the figure you multiplied by, and you have an *improper fraction* equal to the given *whole number*.

Reduce 14 to an *improper fraction*, whose denominator must be 9? *Ans.* $\frac{126}{9}$.

Here because it says 9 must be the denominator, I multiply 14 by 9, and it is 126, which I place over the 9, and it is done. *Ans.* $\frac{126}{9}$.

NOTE

NOTE 1.

To express any *whole number* fraction-wise, it is only putting unity, or 1 under it. Thus, 14, 26, 490, &c. will be $\frac{14}{1}$, $\frac{26}{1}$, $\frac{590}{1}$, &c. Remember this, for it is very useful.

NOTE 2.

Every *improper fraction* is more than an unit, or 1, and every *simple fraction* is less than unity, or 1. Thus, the *simple fraction* $\frac{3}{4}$ of a £. sterling, is but 15 shillings: but $\frac{4}{3}$ of a £ sterling is £1, and $\frac{1}{3}$ over; for if you divide 4 by 3, it is 1, and $\frac{1}{3}$, that is, £1 - 6 - 8.

NOTE 3.

When the numerator and denominator are alike, the fraction is equal to a whole number, Thus $\frac{4}{4}$ is 1, or $\frac{25}{25}$ is equal to 1; because the numerator divided by the denominator produces 1.

CASE 4.

To reduce a compound fraction *to a* simple *one, of the same value.*

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

N. B. N. N. signifies *new numerator*, and *N. D. new denominator*, and *C. D. common denominator*, which pray remember.

1. Reduce $\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{5}{8}$ to a *simple fraction*? *Ans.* $\frac{30}{160}$.

For $2\times3\times5=30$ *N. N.* and $5\times4\times8=160$ *N. D.*

 That

That is, 2 multiplied by 3, and that product by 5, is equal to 30 *N. N*; and 5 multiplied by 4, and that product by 8, is equal to 160 *N. D. Ans.* $\frac{30}{160}$.

*** *Note*, When there are cyphers in the numerator and denominator, then cut them off, and the *fraction* is still the same. Thus, in the foregoing answer the *fraction* is $\frac{30}{160}$ *viz.* $\frac{3}{16}$. So $\frac{4000}{7000}$ is only $\frac{4}{7}$.

2. Reduce $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{11}{12}$ to a *simple fraction*. *Ans.* $\frac{1155}{2880}$.

CASE 5.

To reduce a fraction *to its lowest term, that shall be equal to the original given* fraction.

Divide the numerator and denominator by any figure that will divide them without any remainder; so shall the last quotients be a new numerator, and a new denominator equal to the given fraction.

1. Reduce $\frac{72}{120}$ into its lowest terms? *Ans.* $\frac{3}{5}$.

Divisors, $\overset{2}{\smile}\ \overset{6}{\smile}\ \overset{2}{\smile}$

$\frac{72}{120} \Big| \frac{36}{60} \Big| \frac{6}{10} \Big| \frac{3}{5}$ *Ans.* $\frac{3}{5} = \frac{72}{120}$. So also $\frac{168}{448}$ is $= \frac{3}{8}$, which you may prove by dividing the numerator and denominator by any figure that will divide them both, as was said before.

Note, There is another way to reduce a fraction to its lowest terms at once, and that is by finding a common measurer.

To find a common measure.

Divide the denominator by the numerator, and if any thing remains, divide the last divisor by such remainder, and if any thing again remains, divide the last

last divisor thereby: thus go on till nothing remains, and your last divisor is the *common measure*, or number, that will divide the numerator and denominator, at once into their lowest terms

1 Reduce $\frac{168}{448}$ into its lowest terms. *Ans.* $\frac{3}{8}$.

```
                 168)448(2
                     336
                     ---
                     112)168(1
                         112
                         ---
Common Measure            56)112(2
                             112
                             ---
                               0
```

Here I divide the *denominator* 448 by the *numerator* 168, and 112 remains; then I divide 168, the last divisor, by that 112 and 56 remains; *lastly*, I divide 112 by 56, and 0 remains: so is 56 the *common measure*, that will divide both at one work.

```
56)168(3 N. numerator     56)448(8 N. denominator.
   168                       448
   ---                       ---
     0                         0
```

Ans. $\frac{3}{8}=\frac{168}{448}$.

2 Reduce $\frac{512}{1280}$ to its lowest terms. *Ans.* $\frac{2}{5}$.

CASE 6.

To reduce fractions of different denominators to fractions of the same value, having one common denominator.

First, Multiply all the denominators together for a common donominator; then begin with the first numerator, and multiply it into all the denominators, except its own denominator: do thus with all the other numerators, multiplying them into all the denominators except their own; so will these products be *new* numerators, which must be placed over the *common* denominator,

nominator, and each *fraction* will be equal to *that* fraction whose numerator you multiplied into the denominators.

1. Reduce $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{8}$ to fractions, having one *common* denominator.

First, I multiply all the denominators together, and it makes 120 for a C denominator. That is, $3\times5\times8=120$ *C. D.* Then I begin with the numerator 2, that is, $2\times5\times8=80$ *NN.* Then the numerator $3\times3\times8=72$ *NN.* And *lastly*, $5\times5\times3=75$ *NN.* These new numerators I place over the C denominator, and the work is done. Thus, $\frac{80}{120}=\frac{2}{3}$ for 80 is $\frac{2}{3}$ of 120. Also, $\frac{72}{120}=\frac{3}{5}$ and $\frac{75}{120}=\frac{5}{8}$.

2 Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{4}{5}$ to fractions having one C. denominator. *Ans.* $\frac{720}{960}=\frac{3}{4}$, $\frac{800}{960}=\frac{5}{6}$, $\frac{840}{960}=\frac{7}{8}$ and $\frac{768}{960}=\frac{4}{5}$.

CASE 7.

To reduce fractions *of one name or denomination to another.*

1. ASCENDING.

When a *fraction* is to be reduced from a less to a greater denomination, then make the given *fraction* into a compound *fraction*, by considering how many of the *less* make one of the *greater* denomination; then reduce this *compound fraction* to a simple one, and it is done.

1. Reduce $\frac{1}{4}$ of a penny to the fraction of £. sterling. *Ans.* $\frac{1}{960}$£.

Here, because 12 pence make 1 shilling, and 20 shillings a £. I say, $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$, which reduced to a *simple fraction* by multiplying the numerators together

ther for a *N. N.* and all the denominotors for a *N. D.* gives $\frac{1}{900}$ of a £ $=\frac{1}{4}$ of a penny.

2. Reduce $\frac{3}{4}$ of a farthing to the fraction of a guinea? *Ans.* $\frac{3}{4032\frac{1}{2}}$, *viz.* $\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{21}=\frac{3}{4032\frac{1}{2}}$ *Ans.*

2. Reduce $\frac{1}{4}$ of *lb.* to the fraction of a ton. *Ans.* $\frac{1}{8960}$.

DESCENDING.

When the *fraction* is to be brought from a *greater* to a *less* denomination, then make of it a *compound fraction*, as before; descending from the *great* to *lesser* denomination: then multiply every denominator by the numerator of the given *fraction*, (except its own denominator) and place the product over the denominator to the said given fraction, you have the answer.

N. B. This *case* is a proof to the last.

1. Reduce $\frac{1}{960}$ of a £. sterling to the fraction of a penny? *Ans.* $\frac{240}{960}$, or $\frac{1}{4}$. For $\frac{1}{960}$ of $\frac{1}{20}$ of $\frac{1}{12}$. Now the numerator 1 multiplied into the denominator 20, and 12 is $=240$, which I place over the denominator 960, thus, $\frac{240}{960}$ *Ans.*

2. Reduce $\frac{3}{4032\frac{1}{2}}$ of a guinea to the fraction of a farthing? *Ans.* $\frac{3024}{4032\frac{1}{2}}$. For it is $\frac{3}{4032\frac{1}{2}}$ of $\frac{1}{21}$ of $\frac{1}{12}$ of $\frac{1}{4}$. Now $3\times21\times12\times4=3024$ *N N.*

CASE

CASE 8.

To find the value of a fraction *in* money, weight, *or* measure.

Multiply the numerator of the fraction by the next nearest part belonging to the integer itself, and divide by the denominator; then multiply the remainder by the next nearest part of the integer, and divide by the same divisor, or denominator, and thus go on till you have gone through all the parts of the integer, and the quotients will be your answer.

1. What is the value of $\frac{26}{30}$ of a £. sterling? *Ans.* 17*s.* 4*d.*

```
      26 numerator
      20
     ----
  3|0)52|0
     ----
    17s. 10d.
         12
       ----
    3|0)12|0
       ----
         4d.
```

Here I multiply the numerator 26 by 20, the shillings in a £. and divide by the denominator 30, gives £. 17, and 10 over; which I multiply by 12, and divide again by 30, gives 4 pence; therefore I find $\frac{26}{30}$ of a £. to be 17*s.* 4*d.*

2. What is the $\frac{15}{216}$ of a moidore? *Ans.* 1*s.* 10*d.* $\frac{1}{2}$, as by the following work.

```
      15 numerator
      27 multiply
    ----
216)405(1s.
    216
    ----
    189
     12
    ----
216)2268(10d.
    216
    ----
    108
      4
    ----
216)432(2 qr.
    432
    ----
      0
```

Here I multiply the numerator 15 by 27, the shillings in a moidore, and divide by the denominator, gives 1 shilling, and 189 over, which I multiply by 12, and divide by 216, as before, and so I go on, as appears by the work, and find $\frac{15}{216}$ of a moidore to be 1*s.* 10*d.* $\frac{1}{2}$

Ans. 1*s.* 10*d.* $\frac{1}{2}$.

Tyro. I see the nature of it plainly; and like this *case* very well; for it is diverting as well as improving.

Philo. You are to proceed the same in *weight* and *measure*; therefore I shall only set you a few questions for practice to try at leisure.

3. What is the $\frac{324}{972}$ of a ton? *Ans.* 6 C. 2 *qrs.* 18 *lbs.* 10 *oz.* 10 *dr.* $\frac{648}{972}$.

4. What is the $\frac{14}{112}$ of a barrel? *Ans.* $4\frac{1}{2}$ gallons.

5. What is the $\frac{150}{225}$ of a mile? *Ans.* 1173 yards, 1 foot.

6. What is the $\frac{14}{16}$ of an acre? *Ans.* 3 roods, 20 rods.

7. What is the $\frac{5}{12}$ of a year (allowing 365 days). *Ans.* 152 days, 2 hours.

SEC-

SECTION III.

ADDITION *of* VULGAR FRACTIONS.

Tyro. HOW are *Vulgar Fractions* added together? *Philo.* Very eaſily, by this one rule, *viz.* All *compound fractions* muſt be reduced to *ſimple* ones, and all fractions of different denominators, to a *compound* denominator; then add all the numerators together, and place their ſum over the *common* denominator, is the anſwer; and if it be an improper fraction at laſt, then reduce it to a *mixt* number, and the work is done.

1. Add $\frac{1}{11}$ $\frac{2}{11}$ $\frac{3}{11}$ and $\frac{4}{11}$ together. *Anſ.* $\frac{10}{11}$.

Here being one *common* denominator to all the fractions, I only add the numerators together, and the work is done. *Anſ.* $\frac{10}{11}$.

2. Add $\frac{25}{27}$ $\frac{14}{27}$ and $\frac{19}{27}$ together. *Anſ.* $\frac{58}{27}=2\frac{4}{27}$

Here I find the ſum of all the numerators to be 58, which I place over the denominator 27; but being an *improper* fraction, I divide 58 by 27, and have for *Anſwer* 2 and $\frac{4}{27}$.

3. Add $\frac{2}{3}$ $\frac{3}{5}$ and $\frac{5}{8}$ together. *Anſ.* $\frac{227}{120}$.

Here the *fractions* having different denominators, I reduce them (by *caſe* 5) to a common denominator, and find them to be $\frac{80}{120}$, $\frac{72}{120}$, and $\frac{75}{120}$. Then I add theſe numerators together, and find them 227: ſo the anſwer is $\frac{227}{120}=1\frac{107}{120}$.

4. Add

4. Add $\frac{2}{3}$ of $\frac{3}{5}$ and $\frac{1}{2}$ of $\frac{3}{8}$ together. *Anf.* $\frac{141}{240}$.

Firft, $\frac{2}{3}$ of $\frac{3}{5}$ is $= \frac{6}{15}$, and $\frac{1}{2}$ of $\frac{3}{8} = \frac{3}{16}$. Now $\frac{3}{16}$ and $\frac{6}{15}$ reduced to a *C D* is $\frac{45}{240}$ and $\frac{96}{240}$, which added together is $\frac{141}{240}$. *Anf.*

5. Add. £14 $\frac{3}{5}$ £19$\frac{2}{3}$ £47 $\frac{5}{8}$ and £100 together. *Anf.* £181 $\frac{107}{120}$, or 17*s.* 10*d.*

6. What muft I add to £54 $\frac{2}{3}$, and 19 $\frac{3}{5}$ to make it £100? *Anf.* 25 $\frac{11}{15}$.

SECTION IV.

Subtraction *of* Vulgar Fractions.

Tyro. HOW is *Subtraction of Fractions* performed?

Philo. By the fame Rule as in *Addition*, firft reducing all *compound* to *fimple*; and all to a *common* denominator; thus, *fubtract* the numerator or the *lefs* fraction from the numerator of the other, and place the difference over the denominator, is the anfwer.

From	$\frac{18}{29}$		From	£67 $\frac{21}{47}$
Take	$\frac{15}{29}$		Take	18 $\frac{15}{47}$
Difference	$\frac{3}{29}$	*Anf.*	*Anf.*	49 $\frac{6}{47}$
Proof	$\frac{14}{29}$		Proof	67 $\frac{21}{47}$

Here in both thefe examples I take the numerator of the *lefs* fraction from the numerator of the top, or *greater* fraction, and place the difference over the denominator. Then I prove the work, as in common *Subtraction* by adding the numerator of the difference to the numerator of the *lefs* fraction, and it gives the numerator

tor of the top fraction. And thus for all examples of this ſort.

Tyro. I underſtand you very well, ſir; but ſuppoſe the numerator of the fraction to be ſubtracted be larger than the numerator of the fraction I am to take it out of, how am I to do then?

Philo. This is eaſy enough: for when you cannot take the lower numerator out of the top numerator; then take it out of the common denominator, and to that difference add the top numerator, and place it over the denominator for the true difference; only pray remember this is called *borrowing*, as in common *ſubtraction*, and you muſt carry one to the next figure for ſo doing.

2. Lent	$134\frac{14}{24}$		is=£134-11-8	*The* PROOF *by common* Subtraction.
Received	$93\frac{18}{24}$		is= 95-15	
Due	$38\frac{20}{24}$	*Anſ.*	is=£ 38-16-8	
Proof	$134\frac{14}{24}$		Proof £134-11-8	

3. From $\frac{2}{3}$ of $\frac{3}{5}$ take $\frac{1}{2}$ of $\frac{3}{8}$. That is, from $\frac{6}{15}$ take $\frac{3}{16}$. Now $\frac{6}{15}$ is $\frac{96}{240}$, and $\frac{3}{16}$ is $\frac{45}{240}$. Then I take $\frac{45}{240}$ from $\frac{96}{240}$ and there remains $\frac{51}{240}$. *Anſ.*

4. From 1 take $\frac{4}{11}$. *Anſ.* $\frac{7}{11}$. For I make an *improper* fraction equal to 1, which has a denominator equal to the fraction to be ſubtracted, *viz.* $\frac{11}{11}$ is =1. Then $\frac{7}{11}$ from $\frac{11}{11}$ remains $\frac{4}{11}$. And thus for any other example.

N SEC-

SECTION V.

Multiplication of Vulgar Fractions.

Tyro HOW is multiplication performed?

Philo. The fractions must be reduced to simple ones, as before directed, and mixt numbers to improper fractions. Then the rule is, multiply the numerators together for a new numerator, and the denominators together for a new denominator, and the work is done.

Multiply $\frac{4}{7}$ by $\frac{3}{6}$. *Ans.* $\frac{12}{42}$. For $4\ \ 3=12$ N. N. $7\times6=42$ N. D.

2. Multiply $\frac{247}{6}$ by $\frac{4}{9}$. *Ans.* $\frac{988}{54}=18\frac{16}{54}$.
3. Multiply 4*d.* $\frac{1}{2}$ by 4*d* $\frac{1}{2}$. *Ans.* 20*d.* $\frac{1}{4}$.

First, I reduce $4\frac{1}{2}$ to an improper fraction, which is $\frac{9}{2}$; therefore, I multiply $\frac{9}{2}$ by $\frac{9}{2}$. *Ans.* $\frac{81}{4}=20\frac{1}{4}$, as before.

4. Multiply £4 $\frac{3}{8}$ by £2. Here £4 $\frac{3}{8}=\frac{35}{8}$, and £2 is expressed in fractions $\frac{2}{1}$; therefore, multiply $\frac{35}{8}$ by $\frac{2}{1}$. *Ans.* $\frac{70}{8}=$£8 $\frac{6}{8}$, viz. £8-15.

5. Multiply $\frac{2}{3}$ of $\frac{5}{8}$ by $\frac{3}{7}$ of $\frac{1}{5}$, that is, $\frac{10}{24}$ by $\frac{3}{35}$, *Ans.* $\frac{30}{840}=\frac{3}{84}$. And thus may any sum be multiplied by another, with more exactness than by any other rule, though not so easy as decimal fractions.

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SECTION VI.

Division of Vulgar Fractions.

Tyro. HOW is division of fractions performed?

Philo. First reduce all compound to simple fractions; and all mixt numbers to improper fractions, as before directed: then the rule is, multiply the numerator of the dividend into the denominator of the divisor, for a new numerator, and the numerator of the divisor (or fraction you divide by) into the denominator of the dividend for a new denominator, and you have the answer.

1. Divide $\frac{8}{19}$ by $\frac{5}{12}$. *Ans.* $\frac{96}{95}=1\frac{1}{95}$. Here I multiply 8 into 12, which is 96, for a new numerator, and 5 into 19, which is 95, for a new denominator, which is $\frac{96}{95}$. Here you see the answer is an improper fraction; but if you change the fractions, that is, if $\frac{5}{12}$ were to be divided by $\frac{8}{19}$, then the answer would be $\frac{95}{96}$, which is a simple fraction.

Note, The character ($\div$) signifies division, or that the number is divided by what follows it.

2. Divide £47 $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{5}$ of a £. *Ans.* $\frac{4260}{36}$. For $\frac{1}{2}$ of $\frac{2}{3}=\frac{2}{6}$, and $\frac{2}{3}$ of $\frac{3}{5}=\frac{6}{15}$; therefore, divide £47 $\frac{2}{6}$ by $\frac{6}{15}$, that is. divide $\frac{284}{6}$ by $\frac{6}{15}$. First, $284\times15=4260$ N. N. and $6\times6=36$ N. D. *Ans.* $\frac{4260}{36}$, or £118 $\frac{12}{36}$ = 6*s.* 8*d.*

Tyro. Then I peceive that division of fractions makes more of a sum after divided than before.

Philo. Yes certainly, when the divisor is a simple fraction, or less than unity, as in this case; and it answers the same end as common multiplication, viz. increases the value.

Tyro. Explain this a little more pray.

Philo. Obſerve then the following example.

3. It is required to bring 42 guineas into farthings by diviſion only?

Here by caſe 4, I reduce a farthing to the fraction of a guinea, and find it $\frac{1}{1008}$ for a diviſor. Then I make 42 guineas a dividend, thus, $\frac{42}{1}$. Now, $\frac{42}{1} \div \frac{1}{1008} = 42336$ farthings.

☞ From hence, *Tyro*, ariſes this obſervation: that where any whole number is divided by a ſimple fraction, the quotient will be ſo much larger than the dividend, as the diviſor is leſs than the unity, or 1; but on the contrary, when a ſimple fraction is to be divided by a whole number, then the quotient will be ſo many times leſs than the dividend, as the diviſor exceeds unity.

Thus, £4 divided by $\frac{1}{4}$ of a £. is the ſame to multiply it by £4, viz. £16: but $\frac{1}{4}$ divided by 4 is $= \frac{1}{16}$ only, viz. but 1*s.* 3*d**.

And now, *Tyro*, we are come to that rule wherein all the others are exerciſed, viz.

SECTION VII.

The Rule of Three in Vulgar Fractions.

Tyro. HOW is this rule performed?

Philo. After having reduced compound fractions and mixt numbers as before directed, firſt, (as in the common rule of three) make your firſt and third

* See more of the nature of vulgar fractions in my young algebraiſt's companion, dialogue 3, caſe 3, 4.

num-

number of one name, then multiply your ſecond by your third, and divide by your firſt.

Or rather thus:

Having ſtated the queſtion, and reduced the fractions, multiply the denominator of the firſt number into the numerators of the ſecond and third numbers, for a new numerator, and the numerator of the firſt number into the denominator of the ſecond and third, for a new denominator, and that is your anſwer required.

1. If $\frac{3}{4}$ of a yard coſt $\frac{5}{6}$ of a £, what coſt 51 $\frac{1}{4}$ yards?

$$\begin{array}{r} 4 \\ \hline 205 \\ \hline 4 \end{array}$$

Thus, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{205}{4}$. Now I begin with the denominator of the firſt fraction, thus, $4 \times 5 \times 205 = 4100$ N. N And $3 \times 56 \times 4 = 72$ N. D. So is the anſwer $\frac{4100}{72}$ of a £. = £ 6 18*s*. 10*d*. 2*q*. $\frac{48}{72}$ or $\frac{2}{3}$.

2. If a load of wheat coſt £15 $\frac{2}{12}$ what coſt one buſhel?

Here as 40 buſhels make 1 load, my firſt number will be $\frac{40}{1}$, the ſecond number reduced to an improper fraction is $\frac{182}{12}$ and 1 buſhel will be $\frac{1}{1}$. Then if $\frac{40}{1}$ be $\frac{182}{12}$, what $\frac{1}{1}$. Now $1 \times 182 \times 1 = 182$ N N. and $40 \times 12 \times 1 = 480$ N. D. *Anſ.* $\frac{182}{480}$, which is $= 7$*s*. 7*d*.

Tyro. This is eaſy enough, if this be a general rule.

Philo. It is, and if you look over what has been done, you may with eaſe do any thing in this rule; therefore, I ſhall only ſet you a few queſtions for practice.

 3. What

3. What is the intereſt of £219 $\frac{2}{3}$ for one year, at £5 $\frac{3}{8}$ per cent. *Anſ.* $\frac{28337}{2400}$, viz. £11. 16*s.* 1*d.* 2*q.* $\frac{1920}{2400}=\frac{4}{5}$.

4. Bought South-ſea ſtock to the value of £420 13*s.* 4*d.* and gave £95 $\frac{4}{5}$ per cent. what comes it to? *Anſ.* £402. 19*s.* 11*d.* 2 *q.* $\frac{108}{135}$ or $\frac{18}{25}$.

5. A perſon left 40 ſhillings to 4 poor widows, A. B. C. and D. To A he left $\frac{1}{3}$, to B $\frac{1}{4}$, to C $\frac{1}{5}$, and to D $\frac{1}{6}$, deſiring the whole may be diſtributed accordingly; I demand the proper ſhare of each?

Take $\frac{1}{3}$, $\frac{1}{4}$, &c. of 40 ſhillings, and add them together, it makes but 38 ſhillings: then ſay,

If 38*s.* be $\frac{1}{3}$, viz. 13*s.* 4*d.* what will 40 be? Thus proceed with all their ſhares, you will find A muſt have 14*s.* $\frac{16}{38}$, B 10*s.* 6*d.* $\frac{12}{38}$, C 8*s.* 5*d.* $\frac{2}{38}$, and D 7*s.* 0*d.* $\frac{8}{38}$.

And now, *Tyro*, we will proceed to

DIALOGUE XIII.

SECTION I.

Notation of Decimal Fractions.

Tyro. WHAT do you mean by a decimal?

Philo. Any number, whether cyphers before it or not, having a dot before them, thus, .2 .005 or .4715, &c. are decimals?

Tyro. How are theſe decimals formed?

Philo. Every decimal is a vulgar fraction, having as many cyphers for its denominator, and an unit beſides

ſides. Thus the foregoing decimals expreſſed in vulgar fractions are $\frac{2}{10}$, $\frac{005}{1000}$, and $\frac{4715}{10000}$, &c. but the denominators are caſt away in decimals, becauſe you ſee they are always known, and we work only with the numerator, or the decimal itſelf; and this is the reaſon that decimal fractions are ſo very eaſy to what vulgar are.

Tyro. I underſtand you, but have you no mixt numbers here as well as in vulgar fractions?

Philo. Yes: a whole number, and a decimal after it, is a mixt number. Thus, 47.5 or 5 25 are mixt numbers, ſignifying 47 whole numbers, and $\frac{5}{10}$ parts or $\frac{1}{2}$; and the other is 5 whole numbers, and 25 hundredth parts, or 1 quarter, for 25 is $\frac{1}{4}$ of 100.

Note 1. Cyphers after figures are of no ſignification in decimals, thus, .5000 is but .5 and .750000 but .75, for .500 is $\frac{500}{1000}$, which is the ſame as $\frac{5}{10}$: but cyphers before decimals are neceſſary, for they decreaſe their value. Thus, .5 is 5 tenth parts, or $\frac{1}{2}$, but .05 is but 5 one hundreth parts, viz. $\frac{5}{100}$ in vulgar fractions.

Note 2. Pray remember this: that .5 ſignifies the half of any whole number, thing, or integer. .25 is one quarter, and .75 ſignifies three quarters. For 5 is $\frac{1}{2}$ of 10, 25 is $\frac{1}{4}$ of 100, and 75 is $\frac{3}{4}$ of 100.

This being well underſtood, you may proceed to Addition.

SECTION II.

Addition of Decimals.

Tyro. HOW is addition of Decimals performed?

Philo. The ſame as common Addition, only with this difference, that you muſt ſet tenths under tenths, and hundred parts under hundred parts, not regarding the number of Decimal places, but ſet them all even towards the firſt left-hand row, that ſtands next the whole numbers, and then add them all together as in whole numbers.

One example well explained will be ſufficient, for you will ſee the reaſon more plainly in Reduction,

1. Add £47 .45 7259£5 .25 .000545 and 15 .5 together.

47.45	Here you ſee I place the whole numbers under one another, as in Addition, and the Decimals I place even next the whole numbers, not regarding how far they extend to the right-hand: then I add them as they ſtand, like whole numbers.
.7259	
5.25	
.000545	
15.5	
Anſ. 68.926445	

2. Add .575 .005 .0005 .95 and .3 and 675 together *Anſ.* 2.5055. If you ſet theſe under one another, and add them as before, you will have 2 for a whole number and .5055 a decimal.

Note, Always remember to part the decimals from the whole numbers by a dot. Thus, I find in caſting up this laſt ſum it amounts to 5 figures; but as there are but 4 figures in the largeſt decimal, therefore, I make a dot between the 4th and 5th figure, always keeping an equal number of decimal places.

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SECTION III.

SUBTRACTION *of* DECIMALS.

Tyro. IF Addition of Decimals be performed like whole numbers, I imagine Subtraction is the same.

Philo. It is so, only place the figures to the left-hand, always under one another, let them be cyphers or figures; but don't regard cyphers to the right. An example will make it quite easy.

From	47.0075	From	.7	From	5.25
Take	15.51	Take	.275	Take	.98407
Ans.	31.4975	*Ans.*	.425	*Ans.*	4.26593
Proof	47.0075	Proof	.7	Proof	5.25

Here you see I do by tens, as in whole numbers, and borrow and carry the same, only I never set down cyphers to the right hand of the figures, but entirely discard them.

2. Borrowed 100 guineas, and paid at three several times each £ 27.275 what is to pay? *Ans.* £23.175 or £ 23. 3*s.* 6*d.*

SECTION IV.

MULTIPLICATION *of* DECIMALS.

Tyro. HOW is Multiplication performed?

Philo. Like common Multiplication, having no regard at all to the Decimals till the work is done, and added up: then count how many decimal places you have both in the Multiplicand and Multiplier, and as many decimals as you find in both these, so many figures you are to prick, or point off from the right-hand to the left: so will the figures on the left-hand of the dot be whole numbers, and those towards the left, the decimal parts.

EXAMPLES.

Multiply	7.256	Multiply	.5759
by	.762	by	.0375
	14512		28795
	43536		40313
	50792		17277
Ans.	5.529072	*Ans.*	.02159625*

Here in example 1, I have 6 decimals in the Multiplicand and Multiplier; therefore I prick or dot off 6 towards the left-hand, and have 5 for a whole number, and the rest are decimals.

* *Note,* In example 2, I have 8 decimals, in the Multiplicand and Multiplier; I find upon casting them up, that there are but 7 places of decimals, therefore, I place a cypher before the first figure, and make a dot before the cypher so I have 8 decimal places.

This

This is a standing rule, for had the product been less in the number by 2, 3, or more places, so many cyphers must have been added to supply the deficiency.

Tyro. This is easy enough, as you observed, to what Vulgar Fractions are: and I perceive that money, weight, or measure may be easily multiplied by this rule.

Philo. You say right, for suppose I was to multiply £4 15*s.* by £3 10*s.* it would produce £16 12*s.* 6*d.* For £4 15*s.* is 4.75 (because 15*s.* is $\frac{3}{4}$ of a £.) and £3 10*s.* is £3.5. Therefore £4.75 × 3.5 = £16.625, or 12*s.* 6*d.*

3. Multiply .000075 by .0015. *Ans.* .0000001125.

4. Multiply $4\frac{1}{2}$, viz. 4.5. by 4.5. *Ans.* 20.25, or $20\frac{1}{4}$.

5. Multiply $\frac{1}{2}$ a crown by $\frac{1}{2}$ a crown, at a shilling the integer. *Ans.* 6*s.* .25. viz. 6*s.*—3*d.*

See Multiplication of Vulgar Fractions; and more of the nature of this, with the way and manner of valuing any Decimal, in Reduction of Decimals.

SECTION V.

Division of Decimals.

Tyro. HOW is Division performed?

Philo. The same as common Division; but being a little more difficult than the other rules, you must be the more careful: but if you observe the three following notes, they will help you to work any sum.

Note 1. When there are more decimal places in the Dividend than in the Divisor, then (after the work is done) prick off as many in the quotient to supply that defect; that is, the decimal places of the Divisor, and the quotient will always be equal in number to those in the dividend.

Note,

Note 2. If there be more decimal places in the Divifor than in the Dividend, (before you begin to work) add as many cyphers to the Dividend as make the number of decimal places equal to the Divifor, and proceed as before directed.

Note 3. When it happens that the decimal places in the Divifor and Quotient (after the work is done) are not fo many as thofe of the Dividend; then you muft place as many cyphers before the Quotient, or Anfwer, as will make up that deficiency.

.5)47.3476	.12)63.00	12).063
Anf. 946952	*Anf.* 525.	*Anf.* .00525
See Note 1.	See Note 2.	See Note 3.

☞ Remember that all remainders in Decimals after Divifion are of no fignification, and therefore are taken no notice of.

The fame is to be obferved in long Divifion as in the three foregoing examples in relation of pointing off.

```
47.15)3.747565(0794. Anf.
      33005
      -----
       44706
       42435
       -----
        22715
        18860
        -----
```

But if thus.

.4715)3.747565(7.94. *Anf.*

Here you fee the fame fum is propofed, but the anfwer is different, as you may prove at large; for the figures will be the fame. And thus you fee, let them be whole or mixt numbers the work is the fame.

Tyro. I underftand it: and I perceive it is very eafy to divide money into any number of parts by this rule.

Philo.

Philo. Very eaſy indeed.

6. Divide £42 19*s*. 6*d*. by £4 15*s*. 6*d*. That is divide £42 .975 by £.4.775. *Anſ.*

7. Divide £4.5 (viz. £4 10*s*.) by £3.5 (viz. £3. 10*s*.) *Anſ.* £1.285, or £1 5*s*. 8*d*. $\frac{1}{4}$.

8. Divide 1.753 by £1753. *Anſ.* 001.

This example, though it appears trifling at firſt, may be of ſervice. For ſuppoſe it was required to divide £1.753 among 1753 perſons, the anſwer would be .001, viz. near 1 farthing each. Thus you may vary or alter Decimals at pleaſure; but as for money and the manner of finding its value in any Decimal Fraction, that you will ſee in the next Section, which if you obſerve well, you will ſee the whole order of Decimals, and their relation and harmony, compared with Vulgar Fractions.

SECTION VI.

Reduction of Decimals.

Tyro. WHAT does Reduction of Decimals teach? *Philo.* It teaches to reduce a Vulgar Fraction to a Decimal of the ſame value, and alſo ſhews you how to find that value either in money, weight, or meaſure, &c.

CASE I.

To reduce a Vulgar Fraction to a Decimal.

Add as many cyphers as you pleaſe to the Numerator, (making a dot between the Numerator and the cyphers) then

then divide by the Denominator as in common Division, and making a dot before the figures in the Quotient, you have the Decimal equal to the Vulgar Fraction.

1. Reduce $\frac{3}{4}$ to a Decimal. *Ans.* .75.

$4)3.00$ Here you see .75 is $=\frac{3}{4}$ for 75 is $\frac{3}{4}$ of 100 its proper Denominator.

75.

2. Reduce $\frac{5}{8}$ to a Decimal of the same value.

$8)5.000$ That is, 625 is $\frac{5}{8}$ of 1000, its Denominator.

.625 *Ans.*

3. Reduce $\frac{5}{395}$ to a Decimal. *Ans.* .01265.

4. Reduce $\frac{3}{1753}$ to a Decimal. *Ans.* .00171.

Note, Sometimes in reducing a decimal there will be a remainder; but never regard that, if you have 5 places in a decimal it is sufficient.

CASE 2.

To reduce the known parts of money, weight, or measure to a decimal.

Add cyphers to the lowest denomination (making a dot between the cyphers and the figure, and divide by the parts contained in the next higher denomination; then place the next higher denomination, before that Quotient (with a dot between) and divide by the parts contained in the next higher denomination; and so go on, and your last quotient will be the decimal required.

But this must be illustrated by an example to make it plain.

1. What

1. What is the decimal of 14*s*. 6*d*. $\frac{3}{4}$?

```
   4) 3.00000
   ----------
  12) 6.75000
   ----------
2|0)14.56250
   ----------
    728125 Ans.
```

Here, according to the rule, I add cyphers to the lowest denomination, 3 farthings, and divide by 4; then I place the 6 pence before that quotient, and divide by 12. Lastly, I place 14 shillings before this last quotient, and divide by 20, (viz. by 2) without cutting any figure or cypher off to the right-hand; for there is no occasion for that.

See the proof of this in example 1, next case.

2. What is the decimal of 18*s*. 9*d*. $\frac{1}{2}$. *Ans*. 93958.

Note 1. If you would know the decimal of any number of shillings, from 1 to 19, observe this general rule: if the shillings be even, take the $\frac{1}{2}$ of them is the decimal. Thus the decimal of 16*s*. is .8, and of 18*s*. is .9, &c. But if the shillings be odd, multiply them by 5, gives the decimal. Thus the decimal of 5*s*. is .25; for 5 shillings is $\frac{1}{4}$ of a £. and 25 is $\frac{1}{4}$ of 100. So also, the decimal of 17*s*. is .85, and 11*s*. is .55, and the decimal of 1*s*. is 05; for there must be 2 places when the shillings are odd.

A Rule to find the decimal of shillings, pence, and farthings, at a £. sterling the integer at once.

1. For farthings. Add cyphers to the shillings, and divide by 20.

2. For the pence. Add cyphers to the given pence, and divide by 240, the pence in a £. pricking off according to the rule of division. Thus you will find the decimal of 6*d*. ,025 and of 3*d*. .0125.

3. For

3. For farthings. Add cyphers and divide by 960: thus, the decimal of 3 farthings is 003125.

Note, The ſame is to be obſerved in finding the decimal of weight and meaſure, by adding cyphers to the given denomination, aud dividing by the parts contained in the integer.

To tell the decimal of ſhillings, pence, and farthings, by inſpection.

Note, If the ſhillings be even, take the $\frac{1}{2}$ of them, which will be the firſt decimal figure; then bring the pence and farthings into farthings, and if they be leſs than 25, join them to the firſt decimal figure, ſo have you a decimal of 3 places; but if the farthings be more than 25, ſet down 1 more than they really are; if they be above 40 ſet down 2 more than they amount to; ſo have you the decimal nearly.

1. Let it be required to find the decimal of 14*s*. 6*d*. $\frac{3}{4}$, and 18*s*. 9*d*. $\frac{1}{2}$, as before. Here I ſay the $\frac{1}{2}$ of 14 is 7, the firſt figure; then 6*d*. $\frac{3}{4}$ is 27 farthings, but being above 5, I ſet down 28 farthings by the ſide of the 7, ſo is the decimal .728, as before; and ſo for the other.

2. What is the decimal of 6*s*. 10*d*. $\frac{1}{2}$? *Anſ*. .344.
Here the $\frac{1}{2}$ of 6*s* is 3, and 10*d*. $\frac{1}{2}$ is 42 farthings; but being above 40, I, by the rule, ſet down 2 more, viz. 44; ſo is the decimal .344 nearly.

Note, When the ſhillings are odd, multiply them by 5, and bring the pence and farthings into farthings, as before, and ſet the firſt figure under the ſecond figure of the decimal belonging to the ſhillings, encreaſing them by 1, or 2, as before, you have the decimal.

3. What

3. What is the decimal of 17*s.* and 6*d.* ¾: and 11*s.* 10*d.* ¾?

17 multiplied by 5 is .85		11×5	=.55
6¾ more 1 farthing	28	10*d.* ¾×4×2	= .45
Ans.	.878	11*s.* 10*d.* ¾	= 595

The next case is a proof to this, and more useful.

CASE 3.

To find the value of a decimal in money, weight, and measure.

Multiply the decimal by the parts contained in the integer, and prick off as many figures as there are places in the given decimal, and the figures towards the left hand will be whole numbers, and those that are pricked off are decimals, which decimals only must be multiplied by the next denomination: thus go on, multiplying and pricking off the same number of decimals; so will the figures towards the left hand be the value required.

1. What is the .728125 of a £. sterling?

```
          20
   -----------
s.14.562500
          12
   -----------
    d.6.7500
           4
   -----------
  qrs. 3.00
```

Ans. 14*s.* 6*d.* ¾. See Ex. 1. of last case.

2. What is the .7615 of a guinea?

```
           21
   -----------
         7615
        15230
   -----------
   s.15.9915
           12
   -----------
   d. 11.8980
            4
   -----------
  qrs.3.5920
```

Ans. 15*s.* 11*d.* ¾

3. What

3. What is the .1756 of a ton?

```
        20
   ---------
C .3.5120
        4
   ---------
qrs. 2.0480
       28
   ---------
     3840
     0960
   ---------
lb. 1.3440
       16
   ---------
oz. 5.5040
```

Ans. 3 C. 2 qrs. 1 lb. 5 oz.

4. What is the .09715 of a bar.

```
          36
     ---------
       58290
       29145
     ---------
gallons 3.49740
              8
     ---------
pints  3.97920
```

Ans. 3 gall. 3 pints, and $\frac{9}{10}$, or very near $3\frac{1}{2}$ gall.

To tell the value of any decimal in shillings, pence, and farthings (by inspection only) at a £. sterling the integer.

Note, If there be ever so many places in the decimal, the first 3 figures are sufficient, and all that are required in business.

Note 2. Double the first decimal place towards the left hand, and if the second figure be under 5, then the first figure only doubled will be the shillings; but if the second figure be 5, or above 5, then you must add 1 shilling more to those you doubled; and what remains from the second figure above 5, carry to the next figure, and reckon them as so many farthings; and if above 25, account them still as farthings, only abate 1; but if above 40, then abate 2 farthings less than they really are, and you have the value required.

1. What

1. What is the value of .728125 of a £. *Ans.* 14*s.* 6*d.* $\frac{3}{4}$.

Here I double the first figure 7, which is 14, for the shillings, and then I say, 28 farthings is 7 *d.* but I abate 1, because it is above .25; so it is 14*s.* and 6*d.* $\frac{3}{4}$.

2. What is the .39525 of a £. *Ans.* 7*s.* 10*d.* $\frac{3}{4}$.
Here I say, twice 3 is 6, and the next figure being above 5, I count 1 more, which is 7 shillings; then there is 4 remains from the 9, which I carry to the 5, which is 45 farthings; but being above 40, I abate 2, and call it 43, farthings, which is 10*d.* $\frac{3}{4}$

3. What is the .0672 of a £. sterling? *Ans.* 1 *s.* 4*d.*
Here the cypher doubled is 0; but the second figure being 6, that is 1 shilling and 1 over, which I carry to the 7, is 17 farthings, and abating 1 farthing is 16, or 4 pence; which you may prove by multiplying the decimal by 20, 12, and 4, pricking off as before directed.

EXAMPLES for Exercise.

4. What is the .8145 of a ton. *Ans.* 16 C. 1 qr. 4 lb. 7 oz. 10 dwt. .88.

5. What is the .275 of a lb. troy? *Ans.* 3 oz. 6 dwts.

6. What is the .0729 of a year, at 365 days the integer? *Ans.* 26 days, 14 hours, 36 minutes, 14 seconds

These examples are sufficient for any diligent learner, therefore, I will proceed to put them all in practice.

SEC-

SECTION VII.

The Rule of Three Direct in DECIMAL FRACTIONS.

Tyro. HOW is the Rule of three in decimals performed?

Philo. The same as in common rule of three direct, by multiplying the second number by the third, and dividing by the first.

1. If $\frac{3}{4}$ of a yard cost $\frac{5}{6}$ of a £. what cost $51\frac{1}{4}$ yards? that is, if 3 qrs. cost 16 *s.* 8 *d.* what cost 51 yards, 1 qr.

Decimally thus,

If .75 be .8333, what is 51 .25 yards;
Here multiply your second by the third, and divide by the first, you have the quotient 56.942, viz. £ 56 18 *s.* 10 *d.*
See example 1, rule of three of vulgar Fractions.

2. If a bushel of wheat cost 3 *s.* 9 *d.* $\frac{1}{2}$, what cost 40 bushels, or 1 load? *Ans.* £ 7 11 *s.* 8 *d.*

Decimally thus,

If a bushel of wheat cost .18958, what cost 40?

$$\begin{array}{r} .18958 \\ 40 \\ \hline £\ 7.58320 \end{array}$$

Ans. £ 7 11 *s.* 8 *d*

Tyro. This last example is very short indeed.

Philo. It will always be so when the first number is 1, or unity; for decimals are superior to vulgar fractions for ease and expedition; and though not always so near the truth itself, yet answers every thing near enough for any business or demand. See the next question in variations.

3. If

3. If 5 $\frac{1}{4}$ ells cost £2$\frac{803}{960}$ of a £. what cost 282 $\frac{1}{2}$ ells? That is, if 5$\frac{1}{4}$ cost £2. 16*s.* 8*d.* $\frac{1}{4}$, what cost 282$\frac{1}{2}$ ells?

Decimally thus,

If 5.25 ells cost £2.8364, what cost 282.5 ells? *Anʃ.* £152.625, viz. £152. 12*s.* 6*d.*

4. If a ton of ſweet oil coſt £24-17-4$\frac{1}{2}$, what coſt 14T. 17C. 2qr. 19lb. *Anʃ.* £370.1335, or 2*s.* 8*d.* $\frac{1}{2}$

Now, *Tyro*, in all ſuch ſums as theſe, which either by the rule of three, practice, or vulgar fractions, will become very tedious; yet by finding the decimal of the odd weight, and the odd price, you have only a common multiplication ſum.

That is, multiply 14 ton .88348 by £24. 86875, you will have £370.1335, viz. 2*s.* 8*d.* *Anʃ.*

Note, You may find the decimal of 17C. 2qrs. 19lb. by bringing them into lbs. and dividing by the lbs in a ton, viz. 2240. The ſame for any other weight, or meaſure, by reducing the parts, and dividing by the parts contained in the integer.

DIA-

DIALOGUE XIV.

SECTION I.

SIMPLE INTEREST.

Tyro. HOW is simple interest in decimals performed?

Philo. Very easy by the following rule.

Multiply the principal by the rate per cent. and prick off the decimals gives the interest for one year; and if there be odd time, take the parts of the principal itself, and add to the work, it is done.

Note, For £4 per cent. multiply by .04; for 5 per cent. by .05; for 6 per cent. .06; for 7 per cent. .07.

1. What is the interest of £147-15*s.* for a year, at £4 per cent. per annum? *Ans.* £5-18-2-¼.

£147.75
.04
£5.9100
20
*s.*18)20
12
*d.*2) 40
4
*q.*1|60

2. What is the interest of £275-10*s.* for 1 year, at £6 per cent.

£275.5
.06
£16.530

Ans. £16. 10*s.* 7*d.* ¼

And

And the work will be as easy, if the per cent. be for odd time, as appears by the following examples.

3. What is the interest of £ 1250. 15*s.* for 3½ years, at 6 per cent. per annum?

	£ 1250.75	
	.06	
	75.0450	for 1 year
½	3	
	225.1350	for 3 years.
	37.5225	add for the ½ year.
	262.6575	for 3½ years. *Ans.*

4. A person left his wife by will ⅓ of his estate, which was £ 355 7*s.* 6*d.* Now this lay in hand 3½ years; I demand what interest is due upon it, at £ 4½ per cent. per annum.
Ans. £ 55. 19*s.* 5*d.*

Multiply £ 355.375 by £ 4½, that is by .045 is £ 15 .991875 for 1 year: this multiplied by 35 years gives £ 55.9715625.
And thus you see, that if the principal, interest, or time, be ever so much, yet it is easily performed by decimals.

5. A Merchant made an assurance upon goods in a ship bound to a certain port, to the value of £ 2530, upon condition, that in case of a total loss or damage, the assurers were to pay £ 97½ per cent. deducting ½ per cent. (viz. 10*s.* per cent.) out of it: now the ship was cast away, but there were as many goods saved as amounted to £ 955; I demand what the merchant has to receive? *Ans.* £ 1527. 18*s.* 11*d.*

First

First, £955 taken from £2530, remains £1575 loss. this multiplied by 97.5 is 1535.625: and deducting ½ per cent. from this, is only cutting off 2 figures in the pounds, thus, 15|35, and take the ½ of 15|35.625, which is 7.678125 placing it in the units place of the pounds, and the rest in order, (as follows in division) and subtract it from £1535.625, leaves £1527.946875, viz. 18*s.* 11*d.* *Ans.*

The proof of this is worthy your observation, *Tyro.*

For observe 97½ per cent. wants but 2½ per cent. of £100, that is, of being cent per cent. Now if you multiply £1575 by 2.5 viz. (2½) it gives £39.375, which added to £1535.625, gives the original sum or loss, viz. £1575.

Tyro. I thank you kindly, sir; for this, as you have observed, is a proof to me very plainly.

6. What comes an assurance, or a commission, factorage, or brokerage to, upon £3500, at 18*s.* per cent.

35|00
.9 the decimal of 18*s.*

£31.5 *Ans.* £31.10

And thus decimals perform any thing with ease and pleasure.

Tyro. I see it, sir; and now be pleased to give me an example or two in compound interest.

Philo. I will.

SEC-

SECTION II.

COMPOUND INTEREST.

Tyro. HOW is compound intereſt performed?

Philo. By continual multiplication and addition.

Queſt. 1. What will £100 amount to, if it be forborn 5 years at £5 per cent. per annum, compound intereſt? *Anſ.* £127-12*s.*-6*d* ¼.

1. The common method is to multiply the principal by the per cent. and cut off two figures (which is the ſame as dividing by 100) this gives the intereſt for one year, which is £5; this add to £100, gives £105 for the amount of the firſt year. Then this multiplied by the per cent. gives the intereſt for the 2d year, which added to the principal £105, gives £110.25 for the amount the 2d year, &c. &c. But this is tedious; therefore the beſt way is,

2. Set down the principal 100l. and find the intereſt the 1ſt year and add it to it, and it makes as before 105. Set the 5 under the 100l. and make two dots after it, thus, 5.. this ſaves dividing by 100, and will ſupply the decimal places; then multiply 150l. by 5 (viz. 05 in decimals) keeping always 2 figures of the decimals under the 2 dots, and it produces 5——25, which added to 105l. gives 110.25l. as before, &c. &c. See the operation.

£100 + viz. by .05 is 5.00, viz. 5
5 .. added

Amount		
1 year		105 .. this + .05
	is	5.25 add
2 year		110.25 this x .05
	is	5.5125 add
3 year		115.7625 this x .05
	is	5.7881 add
4 year		121.5506 this + .05
	is	6.0775
5 year		127.6281 *Anſ.* £127.12

Queſt. 2. *A* lent *B* £136.775. which *B* promiſed to pay compound intereſt for, at £6 per cent. per annum, and bound his heirs, executors, &c. to that conditon: Now *A* died, and *B* took no notice of payment till it was at laſt diſcovered by the executors of *A*, that *B* had had the money in hand 20 years: I demand what *B* has got to remit for the debt? *Anſ.* £438.651.

But the ſhorteſt method of all to calculate compound ntereſt is by the following tables.

TA-

TABLE I.

A table ſhewing how much 1 pound ſterling will amount to any number of years under 21, at £5 per cent. per annum, compound intereſt.

Year.	*£5 per cent.*	*Years*	*£5 per cent.*
1	1.05	11	1.7103
2	1.1025	12	1.7958
3	1.15763	13	1.8856
4	1.2155	14	1.9799
5	1.27628	15	2.079
6	1.3401	16	2.1828
7	1.4071	17	2.292
8	1 4744	18	2.4066
9	1.5513	19	2 5269
10	1.6289	20	2.6533

TABLE II.

A table ſhewing how much 1 pound ſterling will amount to any number of years under 21, at the rate of £6 per cent. per annum, compound intereſt.

Year.	*£6 per cent.*	*Years*	*£6 per cent.*
1	1.06	11	1.8982
2	1.1236	12	2.0121
3	1.191	13	2 1329
4	1.2624	14	2.261
5	1.3382	15	2 3965
6	1.4185	16	2.5403
7	1.5036	17	2 6927
8	1.5938	18	2.8543
9	1.6894	19	3.0256
10	1.7908	20	3.2071

1. The explanation of the tables.

The amount of £1 for 1 year, at £5 per cent. is 1.05, this × 105 = 1.1025 the amount of the 2d year; this × 1.05 is 1.1576, &c. &c. The ſame for £6 per cent. which is 1.06 this into 1.06 is equal to 1.1236 for the 2d year, &c. &c.

2. The uſe of the tables.

When any ſum is given for any number of years, then multiply the given ſum by the number given right againſt or anſwering to the number of years, and you have the anſwer at one operation.

Let us take example 1, viz. £100, for 5 years, at £5 per cent.

I look in the table of 5 per cent. and againſt 5 years I find 1.27628, (the amount of £1 for 5 years) this multiplied by £100 gives 127.628, viz. £127-12-6¾ as before.

Tyro. This is ſhort indeed!

Philo. 'Tis the ſame as if it were for 20 years. Thus in example 2, it is required to tell the amount of £136. 15*s*. 6*d*. for 20 years, at £6 per cent. compound intereſt.

I look in the table for £6 per cent. and againſt 20 years I find 3.2071 the amount of £1 for that time; this multiplied by the ſum £136-15 6. (viz. 136.775) gives £438.651, viz. £438. 13 ſhillings and 1 farthing.

Tyro. I like this very well; but ſuppoſe the years are more than in the tables, how then?

Philo. Very eaſy; only add any two or more numbers together, as make the number, and multiply the ſums belonging to each number of years in the table together, gives the right ſum for that number of years.

Thus,

Thus, ſuppoſe I wanted to tell the amount of the laſt queſtion for 30 years, at £6 per cent. Here I take any two numbers, which added, make 30. Suppoſe for example 10 and 20: againſt 10 I find 1.7908, and againſt 20 I find 3.2071, this multiplied together gives 5 7432, the amount of £1 for 30 years, at £6 per cent. which multiplied into £136.775, gives 785.526, the amount of this ſum for 30 years; and thus for any other numbers.

Tyro. I heartily thank you, ſir.

Philo. You ſee how eaſy it is, and this is the only method to calculate annuities, penſions, &c. except you can do it by algebraic calculation.

Note, Theſe tables are eaſily made, the conſtruction depends, as I told you in ſimple intereſt, upon this: That let the per cent be what it will, ſuppoſe £4 per cent, ſay, if £100 be 104, what will £1 be? *Anſ*. £1.04 for 1 year. So alſo for $4\frac{1}{2}$ per cent. it is £1.045; for 5, it is 1.05, &c.

Tyro. I underſtand you very well, ſir, and ſhall endeavour, as time offers itſelf conveniently, to look over theſe things, and make myſelf yet more perfect.

Philo. You will perform your promiſe I hope; but before I leave you, I will give you a notion of the extraction of the ſquare and cube roots, being very neceſſary in many buſineſſes, but eſpecially in the art of menſuation, and ſeveral other branches of the mathematics.

DIALOGUE XV.

SECTION I.

The extraction of the SQUARE ROOT.

Tyro. WHAT do you mean by the words ſquare and root?

Philo. A ſquare number is a number multiplied by itſelf, viz. any figure or figures multiplied by the ſame figure, or figures, the product is the ſquare of that number: thus, $2 \times 2 = 4$, the ſquare of 2; and $9 \times 9 = 81$, the ſquare of 9.

Tyro. This is plain enough. And what is the root then?

Philo. The root is that from which the ſquare is formed: Thus, I told you before, the ſquare of 2 is 4 and the ſquare of 9 is 81; therefore, vice verſa 2 is the root of 4, and 9 is the root of 81, as appears by the following table which ſhould be readily known.

T A B L E.

Roots	1	2	3	4	5	6	7	8	9	10	11	12
Squares	1	4	9	16	25	36	49	64	81	100	121	144

Tyro. How is the ſquare root extracted?

Philo. I will ſhew you the whole proceſs, which pray obſerve.

Suppoſe it were required to extract the ſquare root of 3136, or any other figures.

Firſt,

First, I set down the figures thus, $\dot{3}1\dot{3}6$, and beginning at the units place, I make a dot or point over it, and also over every other figure towards the left hand, as you see in the margin; and pray observe, that as many dots as you have, so many figures the root will always consist of, which here are 2.

Secondly, I seek (by the table) the nearest root to the figures contained in the first point of figures, viz. in 31, and find it to be 5, which I place in the quotient thus, $\dot{3}1\dot{3}6(5$, which figure 5 is called the root or part of the root.

Thirdly, Square the root, that is, multiply it by itself, and place it under the said first points, as in common division, and subtract it therefrom, and bring down the next point, viz. 36, and place it by the side of the remainder, it is 636, which is called the resolvend, as in the margin.

```
3136(5
25
------
 636 resolvend
```

Fourthly, Then I double the quotient figure, or root 5, which is 10, and making another crooked line I place it for a divisor right against the resolvend, thus,

```
           3136(5
           25
           ------
Divisor 10)636 resolvend
```

Fifthly, I now ask (as in division) how many times 10 I can have in the resolvend (always rejecting the last figure) that is, how many times 10 are contained in 63, and find it 6 times, which 6 I put in the root by the side of the 5, and also by the side of the divisor 10, which makes 106; then I multiply 106 by 6, which is 636, and nothing remains: thus I find the square root of 3136 to be 56.

```
    . .
    3136(56 root ans.
    25
    ----
106)636
    636
```

PROOF.

I square the root of 56, that is, I multiply it by itself, viz. 56 by 56, and it gives 3136.

2. What is the square root of 56169?

```
 . . .
56169(237 root Anſ.
4
————
43)161 reſolvend
   129
————
467)3269 new reſolvend
    3269
————
       0
```

Here I proceed the ſame as in example 1, by making a dot over every other figure, and find the nearest the root of the firſt point 5 to be 2, which I ſquare, and place under 5, and remains 1, to which I bring down the next two figures 61, and it is 161; then I double the root 2, it is 4, which I place on the left-hand for a diviſor. Then I aſk how many times 4 are contained in 16, which, tho' it be 4, yet upon trial will be but 3 times (for you muſt obſerve, it will often be leſs then it looks to be) which 3 I place in the quotient, and alſo by the ſide of the diviſor, 4, which makes it 43; then I multiply 43 by 3, and it is 129; and ſubtracting 129 from 161, I have 32 remains, to which I bring down the next point, or two figures 69, and it is 3269, which I call a new reſolvend; then I double the root 23, which is 46 for a new diviſor, and aſk how many times 46 I can have in 326, and find it 7, which I place in the quotient, and alſo after 46, and it is 467; which multiplied by 7, gives 3269. Thus I find the ſquare root of 56169 to be 237. And for a proof I find 237 multiplied by 237=56169. Do you underſtand it, *Tyro*?

Tyro. Very well, ſir: then I perceive that after I have done with the firſt figure in the root, I am to double it, and take down the next point, and then dou-

double the two figures, and take down the next point, and ſo keep on in the ſame order: am I not?

Philo. Your notion is right, and if you conſider well the manner of the working the laſt example, if you have ever ſo many figures, you may do it with eaſe.

Tyro. But ſuppoſe after the work there ſhould be a remainder.

Philo. That matters not at all; only when you come to prove the work, after multiplying the root by itſelf, you muſt add the remainder to the product, and it will be equal to the given number if the work be right.

Sums for practice.

3. I demand the ſquare root of 2996361. *Anſ.* 1731.
4. I demand the ſquare root of 30765 16. *Anſ.* 1754.
5. What is the ſquare root of 43623. *Anſ.* 208, and 359 remains over.

N. B. If you have a mind at any time to know what the remainder will produce, add an even number of cyphers to the ſum, and double the root, and proceed as before: thus, the ſquare root of 43623,000000 is 208,861.

2. To extract the ſquare root of a vulgar fraction: extract the ſquare root of the numerator for a new numerator, and of the denominator for a new denominator.

6, What is the ſquare root of $\frac{25}{81}$? $\frac{5}{9}$.

Note, When you can't extract the root of the numerator and denominator, then reduce the vulgar fraction to a decimal, and extract the ſquare root, you have the anſwer.

7. What is the ſquare root of 1 and $\frac{3}{5}$?

Firſt, $\frac{3}{5}$ is $=.6$ which is 1.6; then I add cyphers, thus, 1.600000, and find the root to be 1.264.

Note, There muſt always be an equal number of decimals.

8. What is the ſquare root of 19.2? *Anſ.* 4.3817.

9. What is the ſquare root of .0003? *Anſ.* .01732.

SECTION II.

The uſe of the ſquare root applied to various branches of the mathematics,

10. I Demand what is the mean proportional number between 30 and 50. *Anſ.* 38.7 tenths.

Multiply one number by the other, and add cyphers and extract the ſquare root, you have 38.7.

11. There is a triangle, whoſe baſe is 30 inches, and the perpendicular 40; I demand the hypothenuſe? *Anſ.* 50.

Note, The perpendicular is that part which is right up; the baſe is that which lies next you, and the hypothenuſe is the ſlanting ſide, called alſo the diagonal line.

A general rule to find the third ſide of any triangle, having two ſides given.

1. Having the perpendicular 40, and baſe 30, as above, to find the hypothenuſe, add the ſquare of the baſe and perpendicular together, and extract the ſquare root of them,

them, you have the hypothenuſe. Thus the ſquare of 30=900. The ſquare of 40=1600, their ſum is 2500, the ſquare root of which is 50, the hypothenuſe required.

2 Having the hypothenuſe and perpendicular, or baſe, given to find the other ſide.

From the ſquare of the hypothenuſe take the ſquare of the baſe, or perpendicular, and the ſquare root is the other ſide required.

EXAMPLES.

12. There is a ſteeple to whoſe vane or top a ſtring is tied, which reaching to the ground, is in length 60 yards, and the diſtance from where it touches the ground to the middle of the ſteeple end is 25 yards; I demand the height of the ſteeple? *Anſ.* 54.5 yards,

Square 60 is 3600, and take the ſquare of 25, viz. 625, from it, and extract the ſquare root, you have 54.5.

13. Suppoſe a ſhip ſails from a certain port, has made 87 miles difference of latitude, and 71 miles departure, what is her diſtance on a regular courſe? This is only finding the hypothenuſe; for add the ſquare of 87 and 71 together, and extract the ſquare root, you have 112.2 miles diſtance.

14. There is a circle or triangle, whoſe ſuperficial content is 30800.25, I demand the ſide of a ſquare, whoſe ſuperficial content ſhall be equal thereto? *Anſ.* 175.5.

15. Suppoſe a rope 5 inches round, I demand the compaſs of another rope, that is double the ſtrength? Square the compaſs of the rope, it is 25, which multiply by 2, and extract the ſquare root, it is 7 07 inches. If it were required to be 3, 4, 5, or 6 times the ſtrength, then multiply the ſquare by 2, 3, 4, 5, or 6, and extract the root.

16. There

16. There is a cable 10 inches round, which weighs 21 C. I demand the weight of one 8 inches round? *Anſ.* 13.44 C.

As the ſquare of the one, viz. 100, is to the ſquare of the other, viz. 64, ſo is the weight of the one to the other, viz. 13.44 C.

17. There is a circle whoſe content is 153.9385, I demand its diameter?

Firſt, as 22 is to 28 :: 153.9385 to the ſquare of the diameter, viz. 195.9217, whoſe ſquare root is 13.99.

And thus I think I have given you ſufficient examples in this rule, that you may with a little more practice become quite maſter of it.

Tyro. I thank you for your care, ſir, and I underſtand it very well; I wiſh I underſtood the cube root as well.

Philo. That you may ſoon do by care, and a little more pains, tho' it be ſomething more difficult than this rule.

DIALOGUE XVI.

SECTION I.

The extraction of the CUBE ROOT.

Tyro. WHAT do you mean by a cube?

Philo. A cube is that which has length, breadth, and thickneſs. Thus ſuppoſe a piece of wood to be cut into the form of a die which is equal every way in length, breadth, and thickneſs, ſuch a figure is called a ſolid, and by name a cube.

Tyro.

Tyro. Give me a further deſcription of a cube in figures.

Philo. You know any number multiplied by itſelf is a ſquare; ſo any number multiplied twice by itſelf is a cube number: thus, the cube of 2 is 8: For 2 × 2 is 4, and 4 × 2=8: So alſo, the cube of 5 is 125: for 5 × 5 × 5=125. Thus you ſee 8 is the cube, and 2 the root of that cube. Alſo 125 is a cube number, whoſe root is 5, as appears by the following table of both ſquares and cubes.

TABLE.

Roots	1.	2.	3.	4.	5.	6.	7	8.	9.
Squares	1.	4.	9.	16.	25.	36.	49.	64.	81.
Cubes	1.	8.	27.	64.	125.	216.	343.	512.	729.

Tyro. How is the cube root extracted?

Philo. To give you a rule for it (I look upon) would be too tireſome for your memory, as there are many parts contained in it: I ſhall therefore take an example or two, and proceed in the whole proceſs, or order of the work.

1. I demand the cube root of 32768? *Anſ.* 32

RULE 1.

Firſt, I make a dot over every fourth figure, beginning at the units place, as in the margin, and as many dots as you have, ſo many places the root will contain, which here are too places, $\dot{3}276\dot{8}$.

Secondly, Seek the root, (or neareſt root) to the firſt point 32, which (by the table) is 3, and place it in the quotient, which is the firſt figure in the root, thus, $\dot{3}276\dot{8}$(3.

Thirdly,

Thirdly, Cube the figure which you put in the quotient (that is, $3\times3\times3=27$) and place it under the first point 32, and subtract it therefrom, thus

```
32768(3
27
-------
 5
```

Fourthly, To this remainder (5) bring down all the figures of the next point (viz. 768) and place them by the side of the remainder, and call this the resolvend, Thus,

```
32768(3
27
-------
 5768 resolvend
```

Fifthly, Triple the quotient, that is, always multiply it by 3) and put the units place of it under the tens place of the resolvend; and call that the triple quotient. Thus,

```
32768(3
27
-------
 5768 resolvend
   9  triple quotient
```

Sixthly, Now square the quotient (that is 9) and triple that square, that is 27, and place the units place of it under the tens place of the triple quotient; that is, place it one figure more to the left-hand, and call it the triple square, thus

Seventhly, Add these two together, and call it the divisor, Thus,

```
32768(3
27
-------
 5768 resolvend
-------
   9  triple quotient
  27  triple square
-------
 279  divisor.
```

Eighty

Eighthly. Ask how many times the divisor is contained in the resolved, rejecting the last figure as you did in the square root; that is, ask how many times 279 you can have in 576 the resolvend, which here is 2, and place this also in the quotient, which now is 32.

```
32768(32
27
------
5768 resolvend
------
   9  triple quotient
 27   triple square
------
Thus, 279 divisor
```

Ninthly, Cube the figure last put in the quotient (viz. 2, whose cube is 8) and place the units place under the units place of the resolvend.

```
32768(32 root
27
------
5768 resolvend
------
   9  triple quotient
 27   triple square
------
 279  divisor
------
   8  cube of 2
```

Tenthly, Multiply the square of the figure last put in the quotient (viz. 4 into the T. quotient, viz. x9. which is 36) and place the product one figure more towards the left-hand.

```
32768(32 root
27
------
5768 resolvend
------
   9  triple quotient
 27   triple square
------
 279  divisor
------
   8 cube of 2
  36 the square of 2 by triple quotient
```

Eleventh.

Eleventhly, Multiply the triple ſquare (viz. 27) by the laſt figure put in the quotient, and place alſo this one figure more towards the left-hand, which is 54

32768(32 root. *Anſ*	
27	
5768	reſolved
9	triple quotient
27	triple ſquare
279	diviſor

Twelfthly, Add theſe 3 laſt numbers together as they ſtand and call it the ſubtrahend, which is 5768 equal to the Reſolvend

8	cube of 2
36	ſquare of 2 by T. qnotient
54	triple ſquare by the root 2
5768	ſubtrahend.

Thus is the work finiſhed, and the cube root of 36768 is found ro be 32.

PROOF.

For the proof of this multiply 32 by 32, and it is 1024, which 1024 I multiply by 32 again, and have 32768.

Note. 1, If the ſubtrahend had been larger than the reſolvend, then I muſt put a leſs figure in the ſecond place in the quotient, and proceed as before directed.

Note 2. When there is another point of figures to take down, firſt, ſubtract the ſubtrahend from the reſolvend, and to the remainder bring down the next point, calling it new reſolvend, or ſecond reſolvend: Then proceed to work as after the firſt reſolvend in every reſpect.

2. Another

2. Another method to extract the cube root, which is in many respects easier and shorter than the former.

Let us take the last example 32768.

First, Find the root of the first point as before, and subtract it therefrom, and to the remainber bring down the next point of figures, and call it resolvend, or dividend, which you please.

Secondly, square the Root, and multiply it by 300 for a divisor; and, as in common division, see how many times it is contained in the dividend, and place it in the quotient, or root, accordingly.

Thirdty, Multiply the divisor by the last figure of the root, and place it under the dividend (units under units) drawing a line between them.

Fourthly, Square the last quotient figure, and multiply it by the first quotient figure, and that product multiply by 30, and set this under the last work, units under units, *&c.*

Fifthly, Cube the last figure and put the units of this under the units of the last, and add these three together in order, as they stand, which is the subtrahend; which if it be more than the resolvend, or dividend, you must put a less figure in the quotient; and proceed as before; but if it be less than the dividend, subtract it therefrom, and the work is done, for two figures in the root: But if there be more figures, bring them down to the remainder, and call it a new dividend; and square the whole root, and multiply by 300, for a new divisor, and put the figures in the quotient. Then square this last figure, and multiply it by the foregoing figures in the root, and then by 30. *Lastly*, Cube the last figure, and place it as be before directed, and the work is done, for three places. The same to be observed for more figures.

Thus,

Thus, the laſt example 32768(32 root. *Anſ.*

27

Diviſor 2700) 5768 dividend

5400
360
8

5768 ſubtrahend,

Tyro. I underſtand it very well, and think this laſt way the eaſieſt.

Philo. Take your choice, as I obſerved before; I ſhall now only ſet you one ſum at large, the firſt way, which you may prove by the ſecond.

2. What

2. What is the cube root of 12812904? *Anſ.* 234.

12812904(234 root.	*Anſ.*
8	
4812	reſolvend
6	triple quotient
12	triple ſquare
126	diviſor
27	cube of 3
54	ſquare of 3 by T. quotient
36	triple ſquare by the root 3
4167	ſubtrahend
645904	new reſolvend
69	triple quotient
1587	triple ſquare
15939	new diviſor
64	cube of 4
1104	ſquare of 4 by T. quotient
6348	triple ſquare by the root 4
645904	new ſubtrahend=new reſolvend.

This you may eaſily prove the other way at leiſure. Alſo 234+234×234=12812904.

QUESTIONS for Practice.

3. What is the cube root of 5396209064? *Anſ.* 1754.

3. I de-

4. I deemand the cube root of 9423479350146861? *Anſ.* 211221.

To extract the cube root of any vulgar or decimal fractions.

5. What is the cube root of $\frac{27}{1331}$? *Anſ.* $\frac{3}{11}$.
6. What is the cube root of $\frac{2197}{9261}$ *Anſ.* $\frac{13}{21}$.

Extract the cube root of the numerator and denominator for a new numerator and denominator.

7. Extract the cube root of 32.768. *Anſ.* 3.2. Proceed as in whole numbers, only prick off as many decimals in the root as you have dots over the decimals.

Note, As in the ſquare root you added either 2, 4, 6, *&c.* cyphers to the decimal, ſo here you muſt add by three's that is, you muſt add either 3, 6, or 9, *&c.* cyphers.

8. What is the cube root of .002? *Anſ.* .1259, and 4383021 remains.

SECTION II.

The Uſe of the CUBE ROOT.

9. THERE is a cube whoſe ſolidity is 1372 feet, I demand the ſide of a cube, whoſe ſolidity is 4 times leſs? *Anſ.* 7.
Divide 1372 by 4, and extract the cube root.

10. If a bullet 2 inches diameter weighs 3 *lb.* what will one of the ſame metal weigh which is 8 inches diameter? *Anſ.* 192 *lb.*

Solids

Solids being in the triple proportion to their ſides or diameter, it is thus found.

As the cube of the given diameter is to the weight, ſo is the cube of the other diameter to the weight of the other required.

12 Suppoſe a ſhot of of 4 inches diameter weighs 18 *lb*. I demand the diameter of another, that weighs 144 *lb*. *Anſ*. 8 inches. This is the reverſe of the laſt queſtion.

13. There is a ſphere, or globe, whoſe ſolid content is 250047 inches: I demand the ſide of a cube, whoſe ſolidity ſhall be equal to the ſolidity of the globe: *Anſ*. 63 inches.

14. A country farmer lent his neighbour out of his hay ſtack 20 feet of hay in length, breadth, and depth, and his neighbour brought it home 10 feet at one time, and 10 at another: how is the balance, and who debtor? *Anſ*. 6000 feet due to him that lent it; he having received but $\frac{1}{4}$th.

15. Suppoſe a ſhip 300 tons burthen, 75 feet by the keel, 29 $\frac{1}{2}$ feet by the beam, and 14 feet deep in the hold; I demand the dimenſions of another ſhip of the ſame make, of 500 tons burthen?

Say, as 300 ton is to 500 ton, ſo is the cube of the given keel to the cube of the ſhip's keel required, the cube root of which is 88.9 feet *Anſ*.

And thus for the other two dimenſions which I leave for your practice.

And now, *Tyro*, before I leave you, I will give you a little hint of meaſuring, gauging, &c. which may poſſibly be of ſervice to you, and your Acquaintance. You muſt expect me to be very ſhort; but you may, by your care and diligence, make a better progreſs.

POST-

POSTCRIPT.

DIALOGUE XVII.

AS this little treatiſe may fall into the hands of ſuch perſons in the country, who would be glad of an opportunity of having a notion of meaſuring a piece of timber, a brick wall, a ciſtern of malt, or a common regular field, or piece of land, *&c.* I have (on purpoſe for their amuſement, and inſtruction of thoſe youth that have a fancy this way) added this poſtſcript, which I make no doubt, will be very acceptable to all ſuch as delight to be induſtriouſly employed at leiſure times: and I perſuade myſelf, it muſt be very agreeable to a parent, in either of theſe ways of life, to ſee his ſon diligent and ready at theſe things; which tho' he may not meaſure ſo exact for want of more learning, proper inſtruments, or experience; yet may come near enough the truth to give ſatisfaction.

Of Flooring, Roofing, *&c.*

Queſt. 1. How many clinkers, 6 inches long, and 3 inches wide, will floor a ſtable 17 feet long, and 9 feet wide? *Anſ.* 1224.

Multiply the length of the ſtable by the breadth, gives 153 feet; this multiply by 144, the ſquare inches in a ſquare foot, gives 22032 inches; this divide by 18, the inches in 1 clinker, gives 1224 *Anſ.*

Queſt.

Quest. 2. How many oaken planks will floor a barn 60 feet $\frac{1}{2}$ long and 33 feet $\frac{1}{2}$ wide; when the planks are 15 feet long, and 15 inches wide? *Anf.* 108.

Multiply $60.5 \times 33.5 = 2026.75$. Then 15 feet $\times 1.25$ feet (viz. 15 inches) gives 18.75 feet for 1 plank: now $2026.75 \div 18.75$, gives 108 planks.

Quest. 3. A thatcher thatches a barn 60 feet long, and 25 feet wide, and the two porches are each 15 feet long, and 10 feet deep, I demand how many ſquares are contained in it? *Anf.* 33 ſquares. *N. B.* 100 feet is 1 ſquare.

Multiply 60×25 gives 1500 feet for 1 ſide, which doubled, gives 3000 for both the ſides; then the porch, viz. 15×10, gives 150 for 1 ſide which doubled, gives 300, which added to 3000 is 3300, which divided by 100 (that is, cutting off two figures) gives 33 ſquares.

2. Of Paving, Painting, Wainſcoting, *&c.*

Tyro. How is paving, painting, and wainſcotting meaſured?

Philo. By the ſquare yard; 9 ſquare feet being 1 yard.

Quest. 4. A gentleman has a walk 22 yards long, and 12 feet wide, which is paved of ſtone; how many yards does it contain? *Anf.* 88 yards.

Firſt, multiply 22 yards, viz. 66 feet, by 12 gives 792, which divide by 9, gives 88 yards.

Quest. 5. There is a room 64 feet round, and 9 feet high, in which are two windows, each 6 feet high, and 3 feet wide, and the fire-place contains 9 ſquare feet; I demand how many yards of paper, half yard wide, will hang it? *Anf.* 118 yards

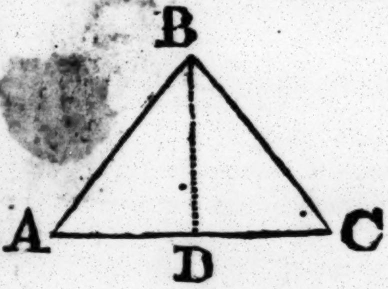
Firſt, $64 \times 9 = 576$ yards, the content out of which take 18 feet, each window, viz. 36 feet, and 9 the fire-place, is 45; and the remainder is 531 feet which divide by 9, gives 59 yards, the content of the room; but as the paper is $\frac{1}{2}$ yard wide only, it will take double this number, viz. 118 yards. *Anf.*

Painting,

Painting, wainſcotting, *&c.* are done by the yard ſquare, and meaſured after this manner.

To meaſure the Peak End of a Houſe or any Triangle.

Queſt. 6. Let *A B C* be the peak end of a roof, whoſe baſe *A C* meaſures 24 feet, and the perpendicular line *B D* from the top of the peak 16, I demand how many ſquare yards it contains?



Multiply $\frac{1}{2}$ the perpendicular *B. D.* by the whole baſe, or line *A. C*; or elſe, multiply the whole perpendicular *B. D*, by $\frac{1}{2}$ the baſe *A. C.* viz. *A. D*, or *C. D*, gives the content in feet, which divide by 100, gives the ſquares, or by 9, gives ſquare yards.

Thus *A C.* 24 feet multiplied by $\frac{1}{2}$ *B. D*, 16 feet, (viz. 8 feet) gives 192 feet, viz. 1 ſquare, 92 feet: or divide by 9, gives 21 $\frac{3}{9}$ ſquare yards of plaiſtering, and thus for any other triangle.

3. Of Board and Timber-meaſure.

1. If the board be regular, multiply the length in inches by the breadth in inches, and divide by 144, gives the anſwer, or, multiply the length in feet by the breadth in inches, and divide by 12, gives the anſwer.

Queſt. 7. There is a board 9 feet long, and 10 inches wide; how many feet does it contain? *Anſ.* $7\frac{1}{2}$ feet.

If the board be wider at one end than another, the true way is to multiply the ends together, and extract the ſquare root for a mean breadth. But cuſtom has brought up an eaſier way, though not ſo true, viz, add the breadth of both ends together, and take the $\frac{1}{2}$ for a

a mean breadth, then multiply the length by this mean breadth into inches (as before directed) and divide by 144.

Queſt. 8. There is a board 16 inches at one end, and 8 at the other, and 10 feet long; what is the content? *Anſ.* By the firſt true way, it is very near $9\frac{1}{2}$ feet? by the cuſtomary way, it is 10 feet.

By the Slip or Sliding RULE.

The Seventh QUESTION proved.

Set the breadth of the board, viz. 10 inches, on the ſlip, to the upper 12 above on the rule; then againſt 9 feet on the rule, you have $7\frac{1}{2}$ feet on the ſlip, the content required.

Again, ſuppoſe a board be 14 feet long, and 15 inches wide, what is the content? Set 15 on the ſlip, againſt the upper 12, then, again 14 feet on the rule, you have $17\frac{1}{2}$ feet on the ſlip, anſwer.

Tyro. I underſtand you, Sir; but pray how do you meaſure timber?

Philo. The cuſtomary way is this: with a ſmall ſtring or cord, take the circumference of the tree (which is done in any place, where the buyer and ſeller can agree) then double this ſtring into 4 parts, and apply it to your rule, and that length is called the girt, or $\frac{1}{4}$ part of the circumference; and it is cuſtomary to abate one inch of the girt on account of the bark.

2. Having got the girt, multiply it by itſelf, that is, ſquare it, and multiply that product by the length of the tree in feet, and divide by 144, gives the content; or multiply it by the length in inches, and divide by 1728, gives the content.

Note, Few perſons mind leſs than $\frac{1}{2}$ a foot in the length of a tree, except it be very large.

Queſt. 9. There is a tree 14 inches girt, and 9 feet long I demand the content? *Anſ.* $12\frac{1}{4}$ feet.

Firſt, 14 multiplied by 14, is 196, this $\times$ 9, the length, $=$1764, which divided by 144, gives 12 feet, 36 inches, which is $\frac{1}{4}$ of 144, viz, 12$\frac{1}{4}$ feet content.

Queſt. 10. There is a tree 10 $\frac{1}{2}$ inches girt, and 12 feet $\frac{1}{2}$ long, I demand the content? *Anſ.* 9$\frac{1}{2}$ feet. For 10.5 $\times$ 10.5 $=$ 110.25 $\times$ 12.5 $=$ 1376.125 $\div$ 144 $=$ 9 57 feet or 9 feet, 82 inches. *Anſ.*

By the Slip, or Sliding RULE.

The Ninth QUESTION proved.

Set the length of the tree on the ſlip (viz. 9 feet) againſt 12 in the middle of the rule (wrote girt line) then againſt the girt (viz. 14) on the girt line, you have 12$\frac{1}{2}$ feet on the ſlip.

The Tenth QUESTION proved.

Set 12 $\frac{1}{2}$, the length, againſt 12, the girt line: then again 10 $\frac{1}{2}$, you have better than 9$\frac{1}{2}$ on the ſlip itſelf, viz. 9 feet, 82 inches.

Of tapering TIMBER.

Some perſons will take but one girt, though a tree be very long and tapering; but this is certainly very wrong, as it may do injuſtice to either the buyer or ſeller. The beſt way is, to meaſure ſuch a tree, as if it were two or three diſtinct trees, by taking two or three ſeveral lengths and girts.

Some, indeed, take 2 girts, one at the great, and the other at the ſmall end, and add them together, and take the $\frac{1}{2}$ of it for a mean girt (as in board meaſure) but this is a hurt to the buyer, and very erroneous; whereas they ſhould multiply one girt by the other, and extract the ſquare root for a mean girt.

Queſt. 11. Suppoſe a tree 20 inches girt, at one end, and 40 at the other, and 9 feet long, I demand the content?

By the cuſtomary way the mean girt will be 30 inches, and the content will be 56 feet, 36 inches $=\frac{1}{4}$

$=\frac{1}{4}$ of another foot. But, according to the true way the mean girt is but 28.28, and the content but 49 98 feet, viz. 49 feet, 141 inches, which is 6 feet 39 inches leſs than the other.

Tyro. This is a ſenſible difference, indeed, in many loads of timber.

Philo. Very true.

Note 1. In ſome countries, 40 feet make a load, and in others, 50 feet make a load.

Note 2. When there are 50 feet to the load, then to caſt up the content, at any given price, the rule is, multiply the content or number of feet, by the price in ſhillings, and cut off the three firſt figures, from the right to the left hand: ſo will the figures towards the left hand be pounds ſterling, and the other will be decimal parts of a £.

EXAMPLE.

Queſt. 12. Suppoſe I meaſured 6 trees, and their content be 548 feet, at 1l. 10s. per load?

I multiply 548 by 30, and gives it 16440, which I cut off thus, 16|440, and it is £ 16.440 viz. 16l. 8s. 9d. $\frac{1}{2}$.

N. B. Stone is meaſured the ſame, only obſerve 8 inches make 1 foot of ſtone.

4. Of BRICK-WORK.

Tyro. How is brick-work meaſured?

Philo. By the ſquare rod, that is, 16 feet $\frac{1}{2}$ in length, and 16 $\frac{1}{2}$ in breadth, make 272 $\frac{1}{4}$ feet, or one ſquare rod; but for common practice, 272 feet only is ſufficient.

Tyro. Is there not a certain ſtandard for the thickneſs of brick-work?

Philo. Yes: all ſorts of brick-work is reduced to the ſtandard of 1$\frac{1}{2}$ brick thick, of which I ſhall give you a further notion by and by.

1. Of Work at $1\frac{1}{2}$ Brick thick the Standard.

Multiply the length by the height, in feet, and divide by 272, the quotient gives the ſquare rods, and the remainder, the feet or parts of a rod.

Queſt. 13. A gentleman built a brick wall round his garden, which was 998 feet long, 9 feet high, and $1\frac{1}{2}$ brick in thickneſs: I demand how many rods it contains? *Anſ.* 33 rods, 6 feet.

Here I multiply 998, the length, by 9, the height, and it gives 8982 feet, which I divide by 272, (the feet in a rod) and it gives 33 rods, 6 feet. *Anſ.*

Tyro. I underſtand it; this is eaſy enough: but ſuppoſe it was but one brick thick, or ſuppoſe it were 2, or 3 bricks thick, how then?

Philo. Having found the content at $1\frac{1}{2}$ brick thick, as before directed, ſay thus:

As 3 (the $\frac{1}{2}$ bricks in the ſtandard meaſure) is to the content in ſtandard meaſure, at $1\frac{1}{2}$ brick thick, ſo is the number of half bricks in the wall to the content at that thickneſs.

Queſt. 14. Let the wall be 998 feet round, and 9 feet high, as before, what is the content, at $2\frac{1}{2}$ brick thick?

The content, at $1\frac{1}{2}$ thick, was found in the laſt queſtion to be 33 rods, 6 feet. Say therefore,

As 3 to 33.6, ſo is 5 half bricks, viz. the thickneſs, at $2\frac{1}{2}$ bricks thick, to the content at that thickneſs, viz. 55 rod, 10 feet.

Tyro. I underſtand you well; but cannot any thickneſs be done at one operation?

Philo. Yes; for having multiplied the length by the height, divide by any of the following numbers, that are ſet againſt the given thickneſs, and you have the content in rods at once, and the remainder is feet.

Note,

Note, Though there be decimals in the diviſors, you may divide by the whole numbers for common uſe.

For	Brick thick divide by	
1		408.3
$1\frac{1}{2}$		272.25
2		204.2
$2\frac{1}{2}$		163.3
3		136.12
$3\frac{1}{2}$		116.6
4		102.1
$4\frac{1}{2}$		90.7
5		81.7

Queſt. 15. There is a wall 15.5 feet long, and 9.5 feet high; what is the content, at 3 $\frac{1}{2}$ bricks thick? *Anſ.* 1 rod, 2 tenths: for I multiply $15.5 \times 9.5 = 147.25$, which divided by 116.6. the diviſor for $3\frac{1}{2}$ bricks, gives 1 rod, 2 tenths. And thus for any thickneſs; for at $4\frac{1}{2}$ thick, it is 1.5 rod, viz. $1\frac{1}{2}$.

By the Slip or Sliding RULE.

There is a wall 9 feet high, and 76 feet long, and $1\frac{1}{2}$ brick thick; I demand the content? *Anſ.* 2 rods, 140 feet, or better than $2\frac{1}{2}$ rods.

Set 272 on the ſlip to the height 9 above it; then again 76 the length on the ſlip, is $2\frac{1}{2}$ or better, on the rule.

A RULE for any Thickneſs.

Set any of the former diviſors, anſwering any thickneſs on the ſlip, to the height; then againſt the length is the anſwer. Thus, the ſame wall at 3 bricks thick.

Set 136 to 9, then again 76 you have 5 rods, the content, ar 3 bricks thick.

5. Of SURVEYING.

Tyro. How is land ſurveyed;

Philo. Land-meaſure is a part of the mathematics, and to ſurvey it true, and in a maſterly manner, you ſhould be provided, 1. With a chain called Gunter's chain. 2. A caſe of inſtruments. 3. A parellel ruler. 4. A plain table. 5. A platting-ſcale, or protracter. And to make it more complete, a theodolite.

Tyro. But cannot I meaſure a common regular field, or little piece of ground, without all theſe inſtruments?

Philo. Yes; by a chain only, or for want of that, a cord, a rod-pole, or any ſuch thing; but this muſt not be depended upon for truth.

Tyro. Give me a deſcription of the chain.

Philo. All land is now generally meaſured by a chain, containing 4 rods, or poles, in length, (viz. 22 yards) according to a ſtatute made in the 33d of Edward I. Anno 1235, which ſays, that a ſquare acre ſhall contain 160 rods, viz. 40 rods in length, and 4 rods in breadth, make 160 rods, or 1 acre of ground.

Note, The Chain is made of iron, containing 100 links, each in length is 7.92 inches, (or nearly 8 inches) 100 of which is 792 inches, or 22 yards, (viz. 4 rods) therefore, 1 chain in length, and 10 in breadth, or 10 in length and 1 in breadth, make an acre.

Note, 2 For want of a chain you may take a cord 22 yards, or 4 rods long, or any number of rods long you pleaſe, dividing it into halves and quarters, with which you may meaſure any common field within a trifle of truth, or, at leaſt, for common ſatisfaction.

Having provided yourſelf with a chain, or any convenient line, if the field or piece of ground, be regular, viz. a ſquare, or the oppoſite ſides alike; then meaſure the length and the breadth, in rods, or parts, and multiply the length by the breadth, and divide the product by 160 the rods in an acre, you have the content.

Queſt. 5. There is a field in the form of a long ſquare (called a parallelogram) whoſe length is 35 rods, and breadth 24 rods, I demand the content in acres? *Anſ.* 5 acres, 1 rood.

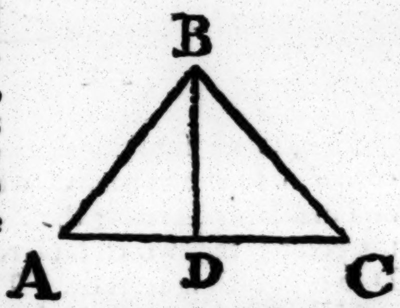
Firſt, I multiply 35, the length, by 24 the breadth, and it gives 840 rods; which I divide by 160, gives 5 acres, and 40 remains, which I multiply by 4 (becauſe 4 roods make 1 acre,) and divide again by 160, gives 1 rood.

Queſt.

Quest. 16. There is a ſquare piece of ground ſet out upon a heath, or common, in order to form a camp for 1000 ſoldiers, each ſide contains 60 rods; how many acres does it contain? *Anſ.* $22\frac{1}{2}$ acres.

For $60 \times 60 = 3600$, which divide by $160 = 22$ acres, 80 rods, or $22\frac{1}{2}$ acres.

Quest. 17. There is a 3 ſided or triangular field, A,B,C, the ſide A,C. is 51.5 rods, and the perpendicular B,D. is 34 rods; how many acres does it contain; *Anſwer* $5\frac{1}{2}$ acres nearly.

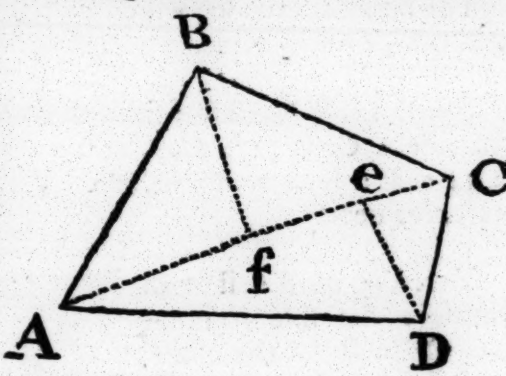


Note, You muſt firſt of all meaſure the ſide from A to C. called the baſe, which ſuppoſe 51 $\frac{1}{2}$ rods; then meaſure half way from A to C, and from D meaſure ſtraight up to the point B, which is called the perpendicular, which ſuppoſe to be 34 rods: now, I told you before, that the baſe multiplied by $\frac{1}{2}$ the perpendicular gives the content; that is, 51.5 multiplied by $\frac{1}{2}$ the perpendicular 17, gives 875.5 which divided by 160, gives 5.47 acres; that is, very near $5\frac{1}{2}$ acres.

To

To measure any four ſided field, whoſe ſides are unequal, called a Trapezium.

Queſt. 18. There is a trapezium, or four ſided field or piece of ground, A, B, C, D, whoſe baſe A C is 64 rods, and the perpendicular B f is 60, and the other perpendicular D e is 40: I demand the content in acres? *Anſ.* 20 acres.



First, to meaſure this field, go ſtraight croſe it from the corner A to the corner C, which here is called the baſe, and meaſures 64 rods: then meaſure right ſtraight from the point B to f, which is 60 rods, and right ſtraight from D to e. which is 40 rods. This done, the rule is,

Multiply the whole baſe A C 64 by $\frac{1}{2}$ B f 60, (viz. 30) and it gives 1920 rods, the content of the triangle A B C: then again multiply the baſe A C by $\frac{1}{2}$ D e 40 (viz. 20) and it gives 1280 rods, the content of the triangle A C D. Add thoſe 2 together, viz. 1920, and 1280 rods, gives 3200 rods, which divide by 160, the rods in an acre, gives 20 for the anſwer.

Tyro. Sir, I thank you; this is enough for my purpoſe at preſent.

Philo. If the field has more ſides, you may meaſure it after the ſame manner, by dividing it into triangles, always remembering to multiply the baſe by $\frac{1}{2}$ every perpendicular that falls upon it.

5. Of

5. Of GAUGING.

Tyro. How may I gauge, or tell the content of any common cooler, or regular caſk, or ciſtern, in gallons, or buſhels.

Philo. I ſhall give you ſome ſhort inſtructions, by which you may tell the content of ſeveral things near enough truth, for your own ſatisfaction: but to be a practical gauger, you ought to underſtand ſeveral branches of the mathematics.

1. To tell the content of a malt ciſtern in gallons and buſhels.

Queſt. 19. There is a ciſtern 6.5 feet long, 4 feet wide, and 3.5 feet deep: I demand its area, and content in gallons and malt buſhels?

Note 1. Area ſignifies the ſuperficial content, or content at one inch deep; which multiplied by the depth, gives the content itſelf.

Note 2. That 282 inches make 1 gallon of ale, water, &c. 231 a gallon of wine, and 2150 inches 1 buſhel, which are your diviſors for all regular figures.

RULE.

Multiply the length, 78 inches, by the breadth, 48, and it gives 3744 inches, which divide by 282, gives 13.276 the area, at 1 inch deep, or divide by 2150, gives 1.741, the area in buſhels. The area multiplied by the depth, 42 inches, gives 557.592 gallons. The area for malt multiplied by 42, gives 73.122, the content in buſhels.

Note, If the area be not required, or you do not underſtand decimals, you may more eaſily find the content

tent at once, thus: multiply the length, breadth, and depth, in inches together, gives 157248, which divide by 282, gives 557 gallons $\frac{1}{2}$, 157248 divided by 2150, gives 73 bushels $\frac{1}{10}$ as before.

To find the area by the sliding rule.

Set 282 upon B to 48, the breadth on A; then against 78 the length on B, is 13.277, the area in gallons.

For malt, set 2150 on B to 48 on A; then against 78 the length on B, is 1.741 on A; the area in bushels: and thus for any regular figure.

To gauge a tub or cooler in the form of a cylinder, viz. whose top and bottom diameters are equal.

RULE.

Square the diameter, viz multiply it by itself, and this product by the depth, then divide by 359 for beer gallons, 294 for wine, and 2737, (or rather by 2737.47) for malt.

Quest. 20. There is a tub 4 feet 2 inches diameter, and 3 feet 4 inches deep: I demand the content in beer, wine, and malt?

I square the diameter 50, which is 2500, and multiply this by the depth, 40 inches, gives 100000; which divide by 359, gives 278 ale gallons; divide by 294, gives 340 wine gallons, and by 2737 only, gives 365 malt bushels.

Note, If the figure be the form of a triangle, or trapezium (as in *questions* 17 and 18) you must proceed to measure them as there directed, and after having multiplied by the depth, divide 282 for beer, 231 for wine, and 2150 for malt, gives the content.

Quest.

Queſt. 21. There is a tub, whoſe top diameter is 40 inches, bottom 30 inches, and the depth 60 inches: I demand the content in beer, wine, and malt?

There are ſeveral ways to do this. One is this, multiply the diameters together and extract the ſquare root for a mean diameter, which is here 34.64. This multiplied by itſelf, and divided by 359, gives the content in gallons, or by 2737, gives the malt buſhels.

Or more eaſily thus, though not ſo true.

Add the diameters together, and take the $\frac{1}{2}$ for a mean, is 35. Now $35 \times 35 = 1225 \times 60 = 73500$, which divided by 359, gives about 205 gallons; and ſo for malt, viz. gives 26 buſhels $\frac{8}{10}$.

For a couch of malt.

1. If it be a regular ſquare only, multiply the length, breadth, and depth together, and divide by 2150, gives the buſhels.

2. If it be a triangle, or trapezium, proceed as before directed, and divide ſtill by 2150.

Tyro. But ſuppoſe the couch be uneven, how ſhall I tell where to take the depth?

Philo. Take the depth at 4 or 5 places, add them all together, and divide by the number of places you took the depth at, for a mean depth.

Queſt. 22. There is a bed or couch of malt, in the form of a long ſquare, whoſe length is 35 feet, breadth 16 feet, and I find the mean depth to be 8.5 inches. viz. $8\frac{1}{2}$ inches; I demand the content?

Thus, 420 inches $\times 192 = 80640 \times 8.5 = 685440$; this divided by 2150, gives 318.8 buſhels.

Of

Of cask gauging.

There is a great variety in gauging casks; but the following methods will be near enough truth for all common casks; such as barrels, butts, &c. that are pretty much bulged.

First, Having taken the bung and head diameters, the rule is, to the sum and $\frac{1}{2}$ the sum of the squares of the bung, and head diameters add $\frac{1}{2}$ the difference of the said squares: this sum multiply by the length, and divide by 1077 for beer, or 882 for wine gallons.

2. *Rule*, which is as true, and much easier.

To the double square of the bung diameter add the square of the head diameter; then multiply this sum by the length of the cask, and divide by 1077 for beer, or 882 for wine.

Quest 23. There is a cask, whose bung diameter is 28 inches, head diameter 25 inches, length 36: I demand the content in ale gallons?

First, The square of the bung diameter 28 is 784; which doubled is 1568. Then the square of the head, viz. $25 \times 25 = 625$, which added to 1568, is 2193; this $\times$ 36, the length is 78941, which divided by 1077, gives 73 gallons, 2 pints for beer, and divided by 882, gives 89 gallons, and $\frac{1}{2}$ wine or brandy.

Note, If you find the area of the bung, and head diameters (by *question* 20,) and add twice the area of the bung, viz. 2.184 to the area of the head 1.741, it is 6.109, which multiplied by $\frac{1}{3}$ of the cask's length, viz. 12, gives 73 308 gallons as before.

These methods holding good for most casks, I shall give no more examples.

Note 2. If one of the head diameters be larger than the other, and the cask is straight in the sides, like some churns, then (by *question* 21) find a mean diameter throughout, and proceed as therein directed.

6. Of

6. Of Cross Multiplication.

There are two methods.

1. By multiplication only.

1. *Rule.* Feet multiplied by feet produce feet.
2. Inches multiplied by feet, or feet by inches, produce inches.
3. Inches multiplied by inches produce parts.

Note, 12 seconds make 1 part, 12 parts make 1 inch, and 12 inches 1 foot.

2. By multiplication and division.

Rule 2. Having placed the lesser sum for the multiplier, multiply the very last place of the multiplicand towards the right hand by the first place or name of the multiplier, and carry one for every 12, setting down what is over 12 under the part you multiplied, then take the parts of the multiplier as in practice, carrying as before 1 for every 12.

But an example will render it more easy, if I give it both ways.

	Feet	*In.*	*Parts*		*Feet*	*In.*
Multiply	4	3	6	*by*	4	3
by	4	3				
	17	2	-			
	1	-	9			
			1	6		
Ans.	18	2	10	6		
	F.	*In.*	*P.*	*Sec.*		

First, I begin and multiply the top 4 feet, 3 inches, and 6 parts, by 4 feet, (carrying 1 for every 12) ſaying, 4 times 6 is 24 parts, that is, 0 and carry 2; then 4 times 3 is 12, and 2 I carried is 14 inches, that is 2 inches, and carry 1 to the feet; then 4 times 4 is 16 feet, and 1 is 17.

Secondly, I multiply now 4 feet, 3 inches, 6 parts, by the lower 3 inches, ſaying 3 times 4 is 12 inches (becauſe feet multiplied by inches are inches) then 3 times 3 inches is 9 parts (for inches by inches produce parts) and laſtly, 3 times 6 parts is 18 parts, viz. 1 part, 6 ſeconds.

The ſecond method.

I firſt multiply the firſt or top line as before, and find it as before, 17-2-0; and now I take the parts as in practice; ſaying, 3 inches is $\frac{1}{4}$ of a foot, &c. See the work.

		Feet	*In.*	*P.*	
		4	3	6	
		4	3		
In.		17	2	-	
3	$\frac{1}{4}$	1	-	10	6
		18	2	10	6 as before.

I ſay 3 inches is $\frac{1}{4}$ of a foot, and take the parts from 4 feet, 3 inches, 6 parts, ſaying, the 4th of 4 is 1 foot; then the 4th of 3 inches 6 parts, (viz. 42 parts) is 10 times (4 is 40) and 2 parts over; laſtly, I ſay the 4th of 2 parts (viz. 24 ſeconds) is 6 ſeconds; which is now done in 2 lines only.

And now, *Tyro*, I muſt bid you farewell, and I hope you will take care to improve yourſelf in them, rather than trifling away your time with idle fancies: for it is evident, that the knowledge of arithmetic is ne-

ceſſary

ceſſary in every ſtation of life, ſince, almoſt all manner of buſineſs depends upon it: and not only this, *Tyro*, but it is a great help to protect us againſt the frowns of fortune, and keep us (by being qualified for ſome lawful poſt, or employment) from thoſe common temptations and misfortunes, to which thoſe, who know the want of it, ſo often fall into, and pay for ſo dearly.

Tyro. I return you thanks for this advice, and hope I ſhall make ſuch uſe of it as may not fruſtate your good deſigns.

Philo. I make no doubt but you will. And therefore ———I once more bid you an hearty farewell.

Tyro. Sir, I am your humble ſervant.

F I N I S.